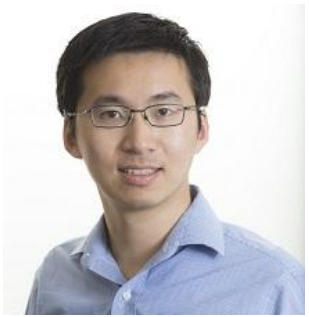
Fully Dynamic Electrical Flows: Sparse Maxflow Faster than Goldberg-Rao

Yu Gao, Yang P. Liu, Richard Peng

Georgia Tech

Stanford University

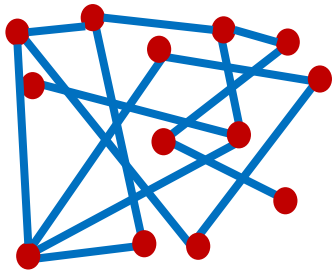


Georgia Tech &
University of Waterloo

Talk Organization

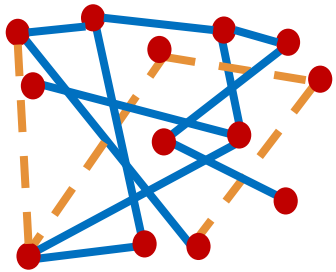
Flow Problems

Part 1



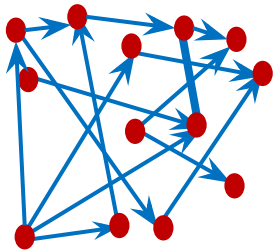
Fully Dynamic Electric Flows

Part 2



Putting it Together: Full Algorithm

Part 3



The Maximum Flow Problem

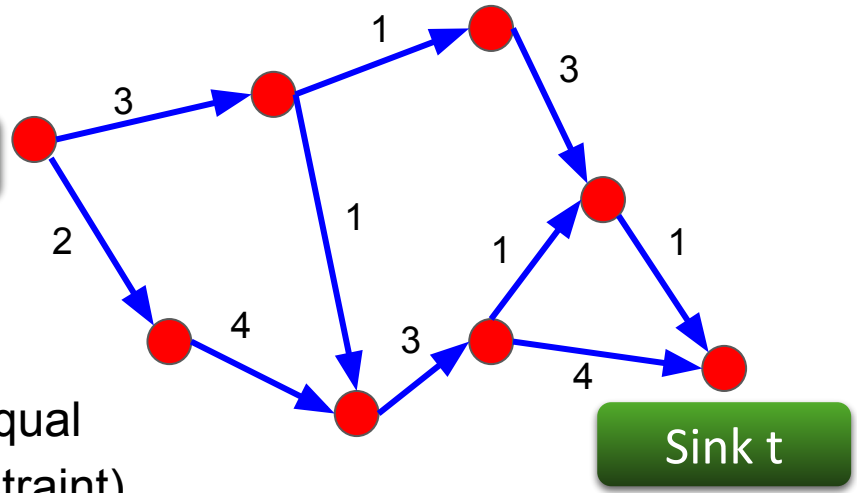
Directed graph G = (V, E) with n vertices and m edges.

Edge e has capacity u_e in [1, 2, …, U].

Source vertex s and sink t.

Route as much flow from s -> t such that flow on edge e in between [0, u_e].

Also, every vertex except s, t must have equal incoming and outgoing flow (demand constraint)



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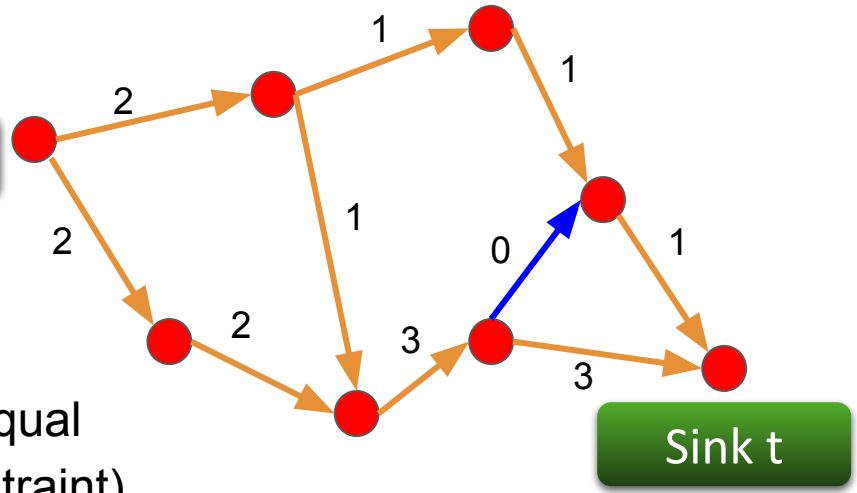

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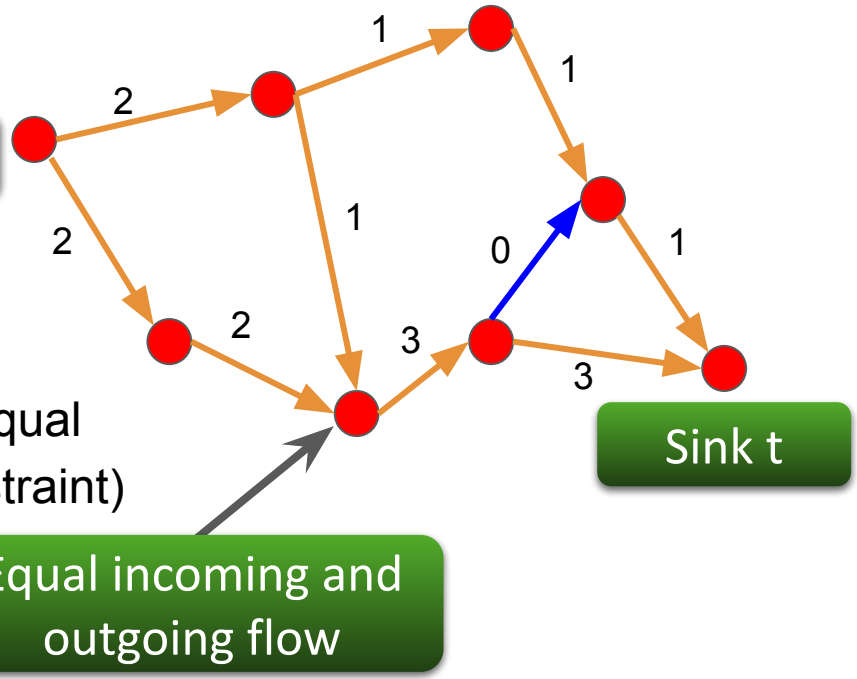

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Let f in R^E denote the *flow vector*, i.e. f_e = flow on edge e.

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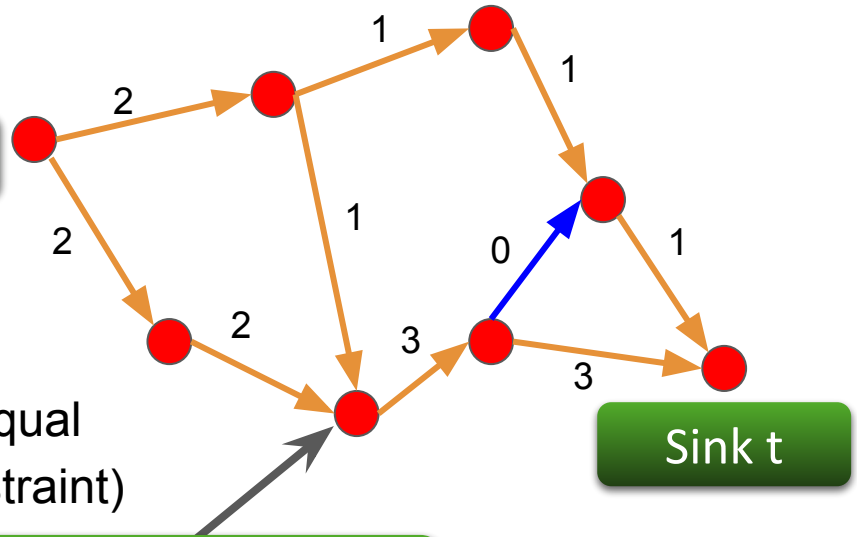

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Fundamental problem

Well studied with decades of extensive research

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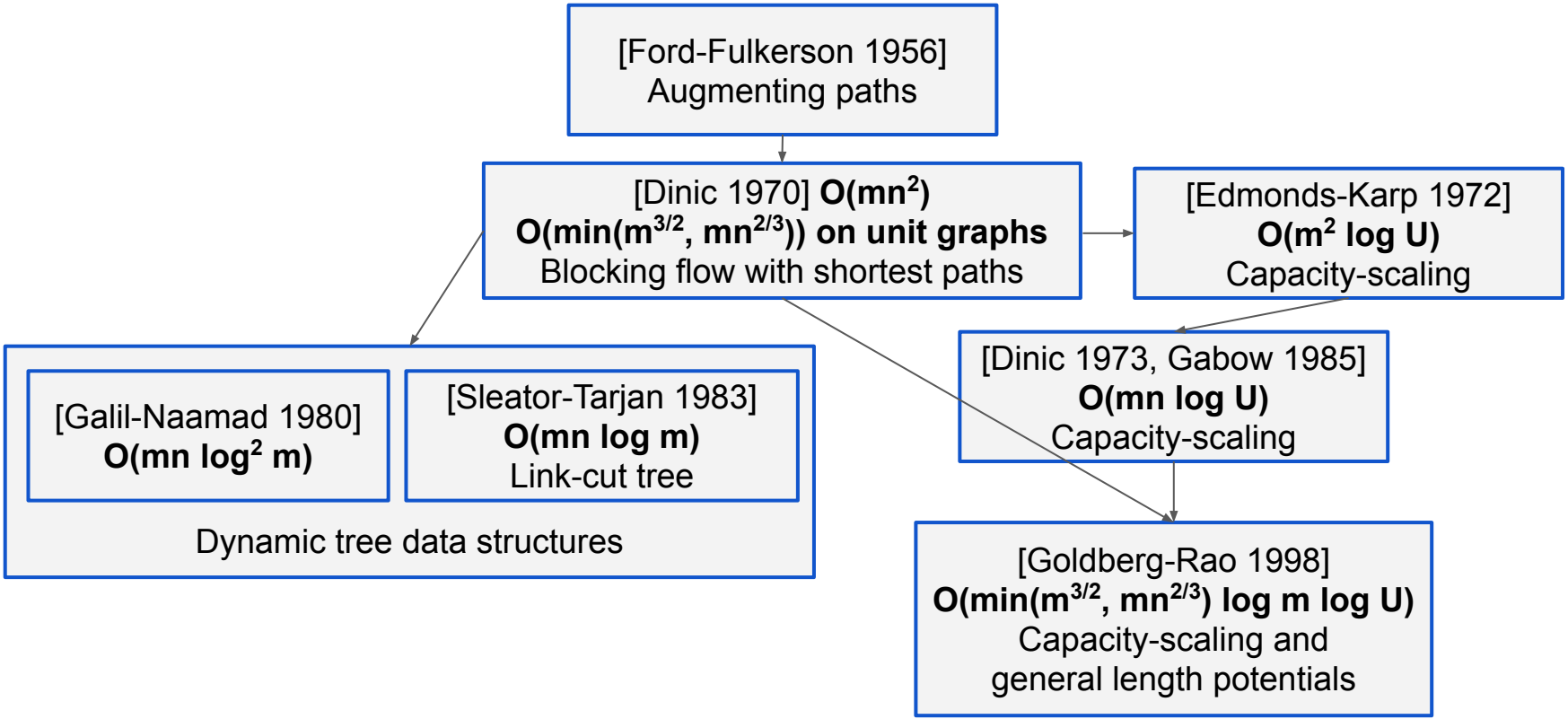
Applications

Minimum s-t cut, bipartite matching, scheduling, transportation, clustering

Methods often solve more general cases: mincost flow, negative weight shortest paths, mincost matching, optimal transport

Previous Work on Maximum Flows: Classical Results



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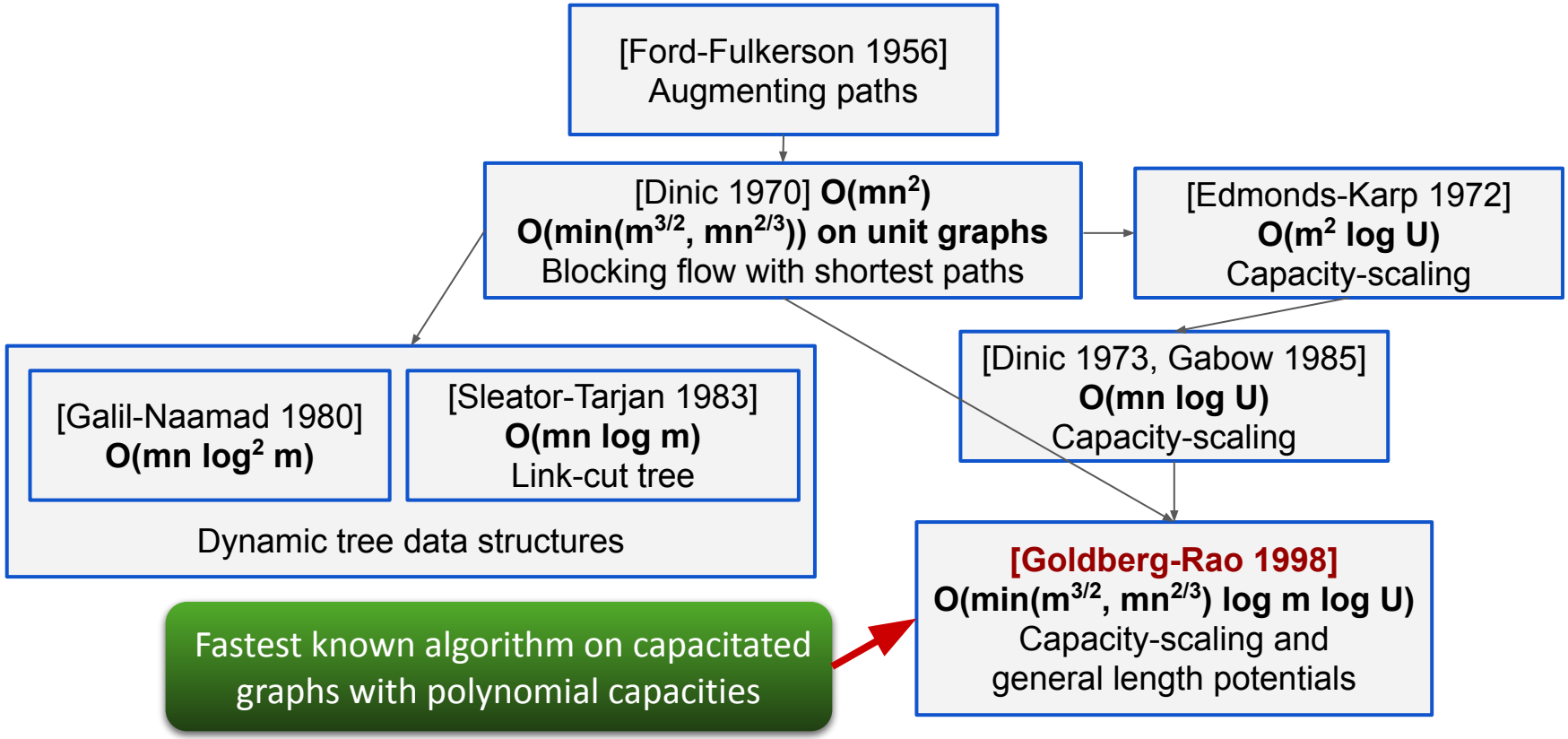

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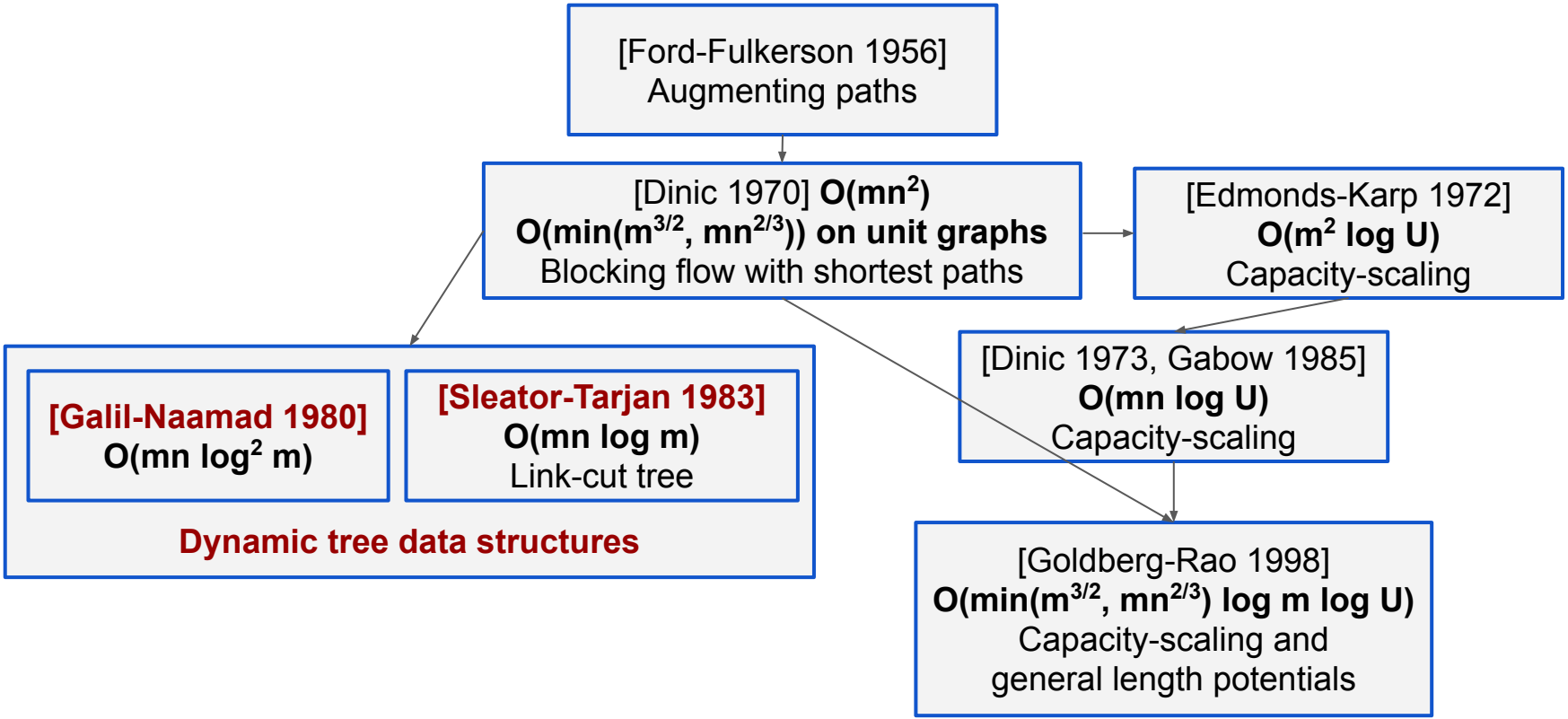

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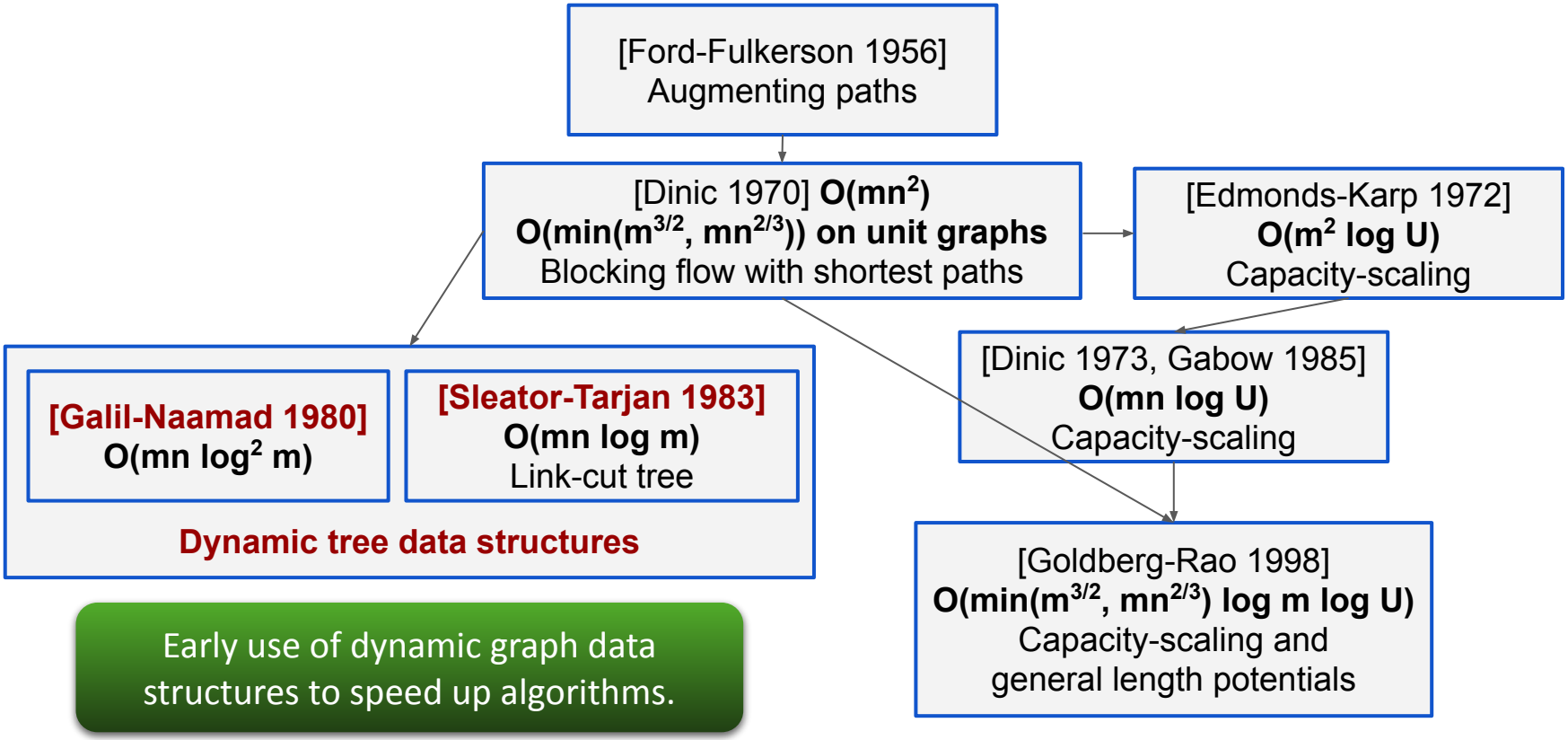

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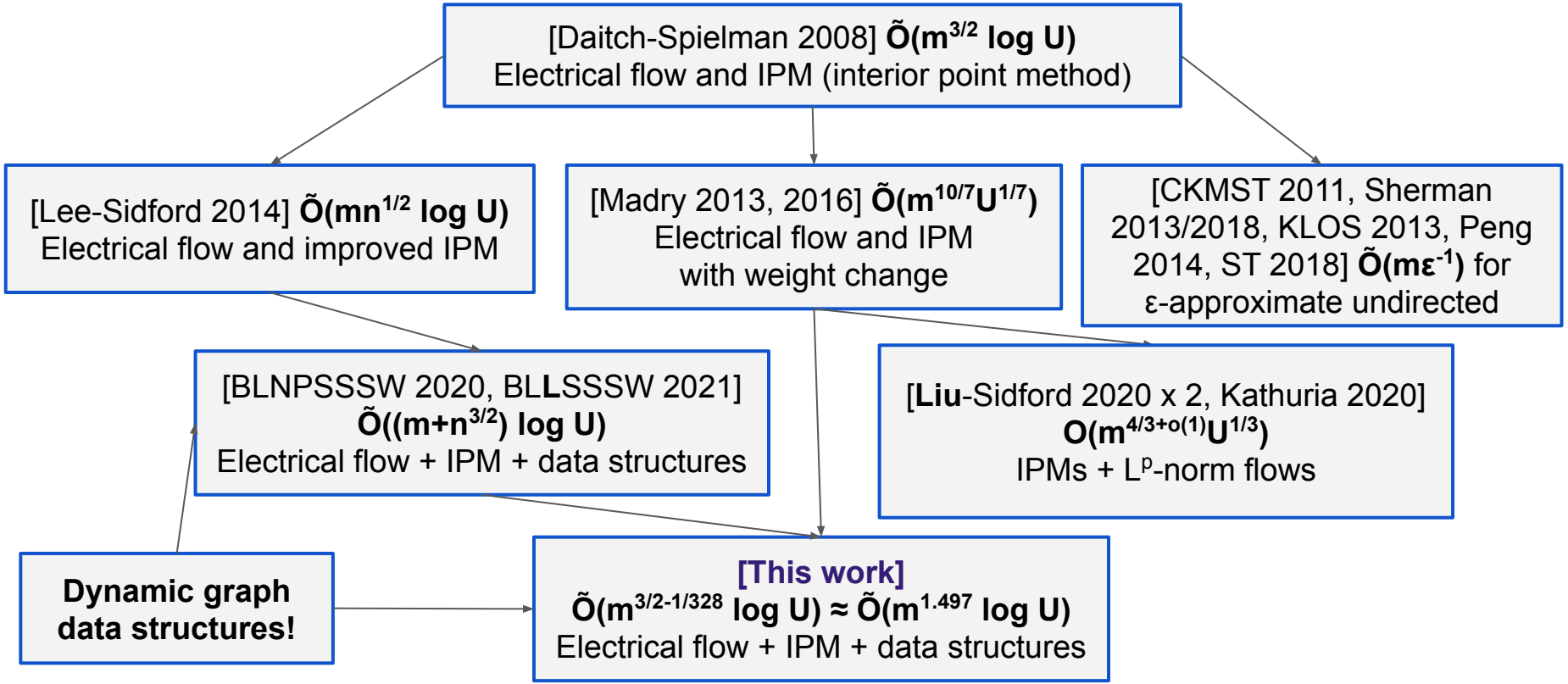

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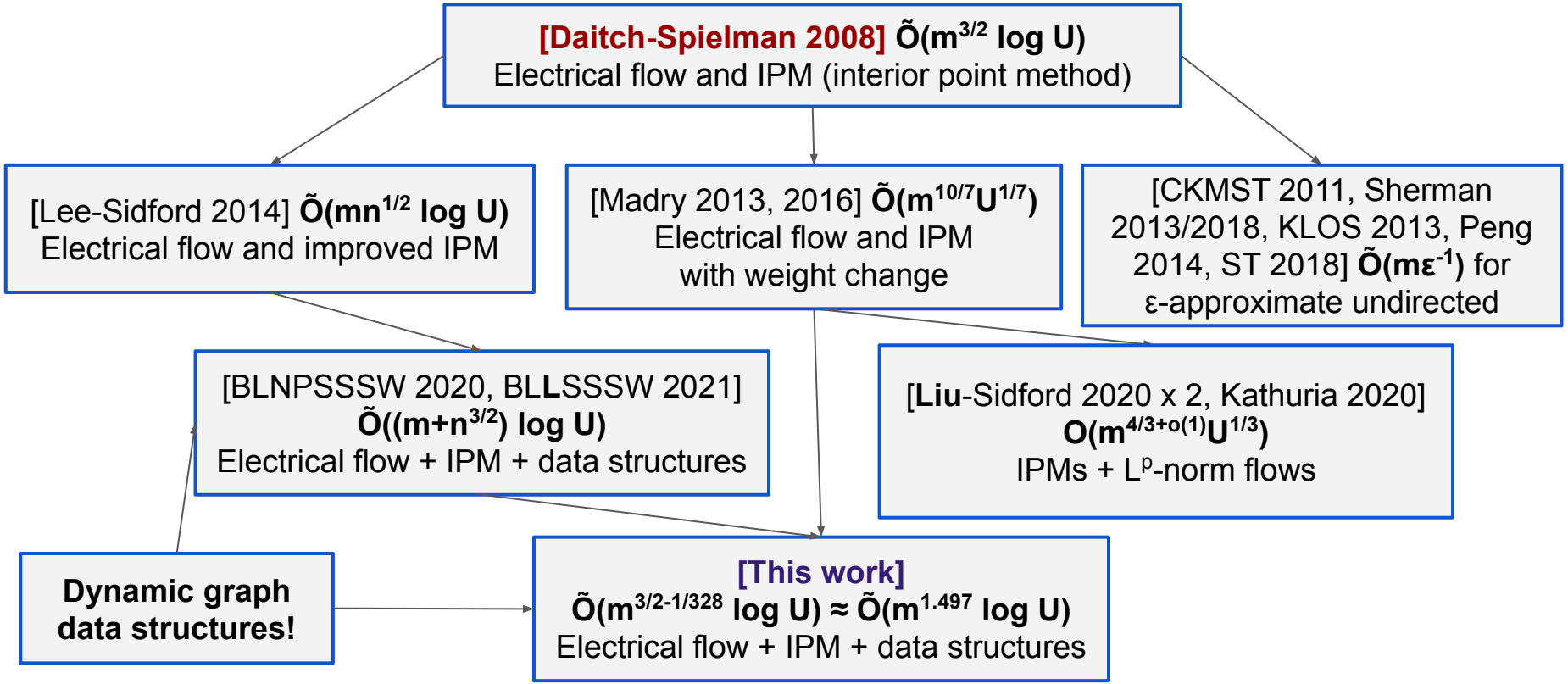

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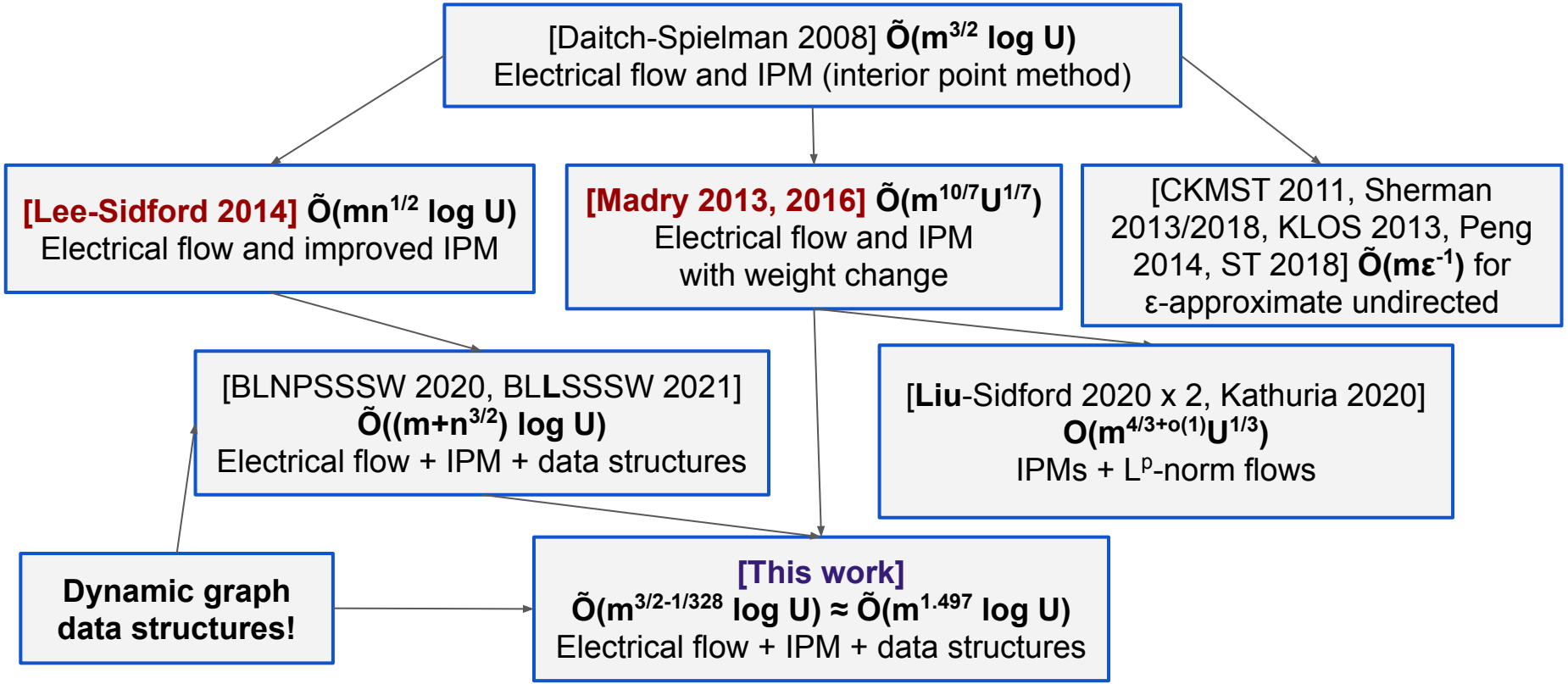

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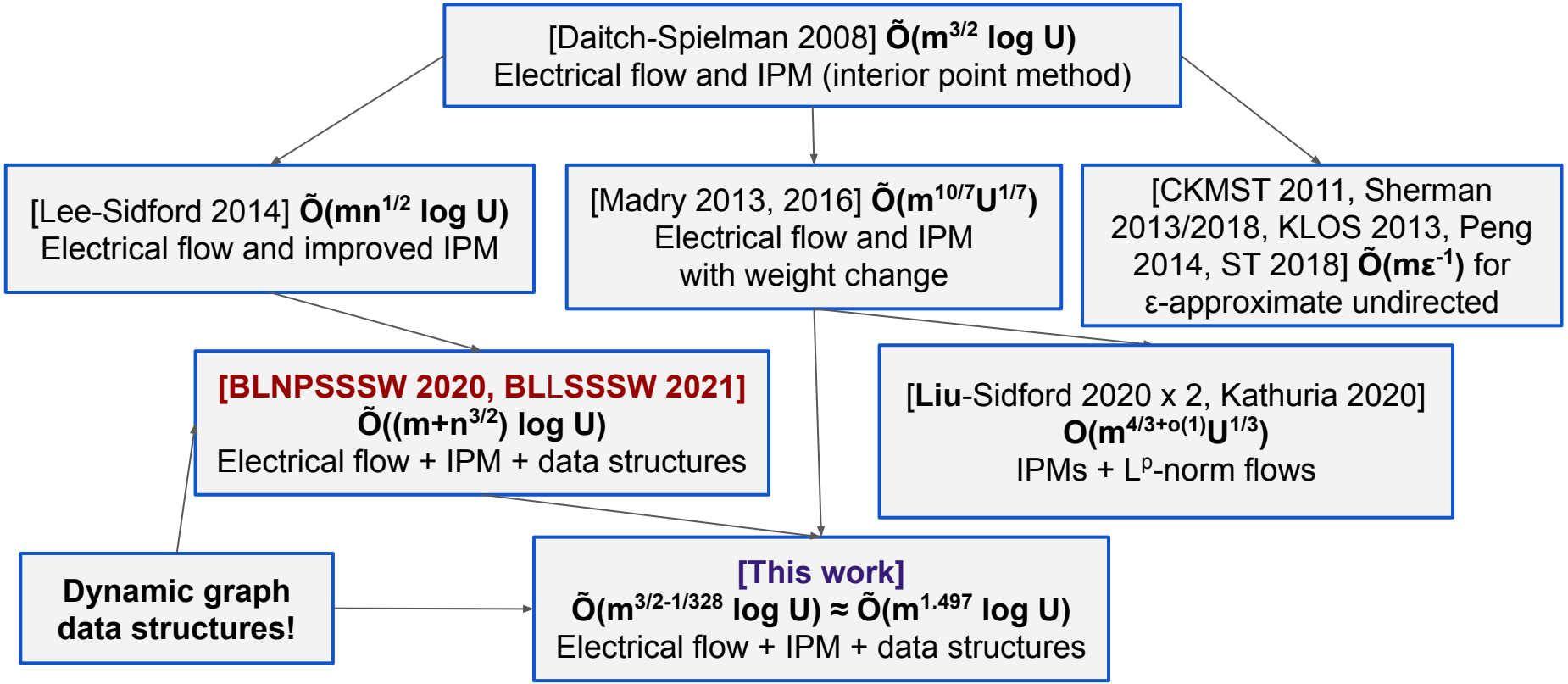

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Undirected graph G = (V, E), edge e has *resistance* r_e.

s-t electric flow is the minimum energy flow, i.e. it minimizes $\sum_e r_e f_e^2$ over all flows sending one unit from s to t. $\sum_e r_e f_e^2$ is known as the *energy.*

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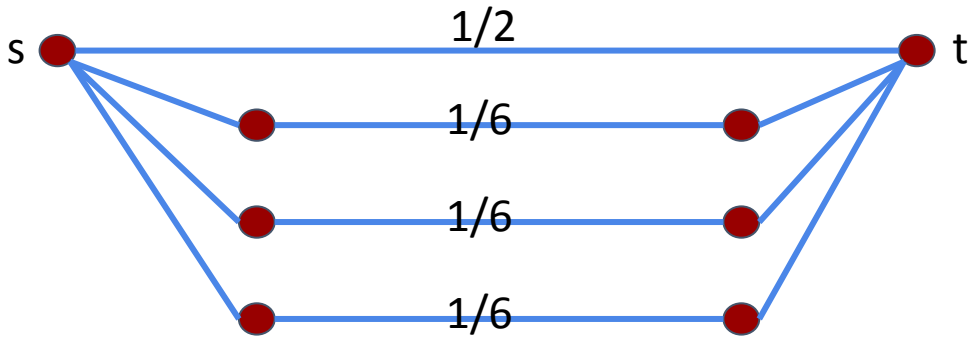
Physically, corresponds to an electrical circuit where edge e is a resistor with resistance r_e, and a battery is hooked to s, t to induce voltage drop.



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Basic approach: Build maximum flow from $\tilde{O}(m^{1/2})$ electrical flows.

Question: Less than O(m) per iteration?

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Idea 1. Change method to reduce number of iterations: $m^{1/3+o(1)}$ for unit capacity graphs [Madry 2013/2016, **Liu**-Sidford 2020 x 2, Kathuria 2020].

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At a high level, such a data structure with sublinear m^{1-c} amortized runtime improves over $O(m^{3/2} \log U)$ time maxflow.

Our Results: Heavy Hitters for Dynamic Electric s-t Flows

Randomized data structure against oblivious adversaries.

Guarantee: The energy of the s-t electric flow is <= 1 at all times.

Operations:

- Initialize(G = (V, E), r in R^E, ε) -- Graph G, resistance r_e on edge e, accuracy ε
- Update(e, r^{new}) -- changes the resistances of edge e to r^{new}.
- Locate() -- Returns $O(\varepsilon^{-2})$ edges containing all edges e with *energy* $r_e f_e^2 >= \varepsilon^2$. (Here f_e is *any* (1+ε)-approximate s-t electric flow)

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Theorem **[GLP21]**: There is a data structure supporting these operations in armotized sublinear $m^{0.99}\varepsilon^{-O(1)}$ time per query.

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Theorem **[GLP21]**: Maximum flow on graphs with capacities in [1, U] can be solved in time $\tilde{O}(m^{3/2-1/328} \log U)$.

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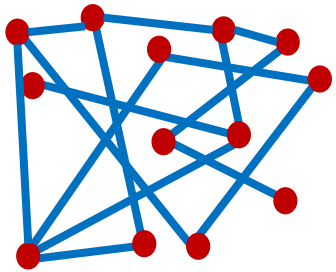
Approach: use the previous data structure to implicitly add $\tilde{O}(m^{1/2})$ electrical flows to get the final maxflow, implementing each iteration in sublinear time.

Talk Organization

Flow Problems

Part 1



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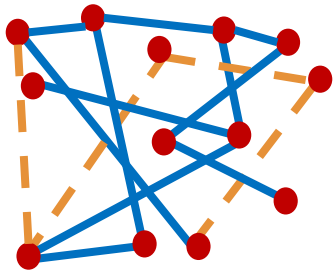

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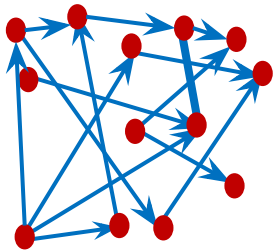

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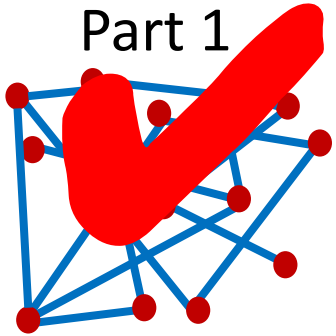

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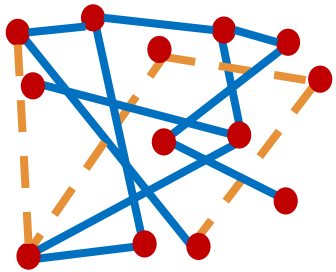

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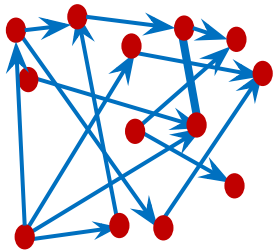

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Goal: Build data structures that achieve efficient (eg. sublinear) update time for processing graphs that are changing.

Examples: edge connectivity, flows, shortest paths and reachability in directed graphs, effective resistances

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Approaches: Graph decompositions -- reduce the problem on a large graph to a similar problem on a smaller graph

Referred to as *vertex sparsification*: reduce the number of vertices while maintaining important quantities, i.e. edge connectivity or effective resistance.

Previous Work on Dynamic Graph Data Structures

Edge connectivity: under a graph with edge insertions and deletions, decide whether vertices s-t are c-edge-connected, i.e. there are c disjoint paths.

[Fre85, GI91b, GI91a, WT92, EGIN97, Fre97, HK97, HT97, HK99, Tho00, HdLT01, KKM13, Wul13, KRKPT16, NS17, NSW17, Wul17, HRT18, CGL+19].

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Dynamic effective resistances: [DGGP19, CGHPS20] sublinear time (1+ε)-approximate pairwise effective resistance in dynamic graph.

Effective Resistances and Schur Complements

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Laplacian: For R = diag(r) (diagonal matrix of resistances), $L = B^T R^{-1} B$.

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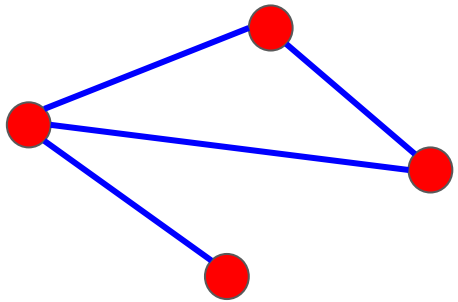

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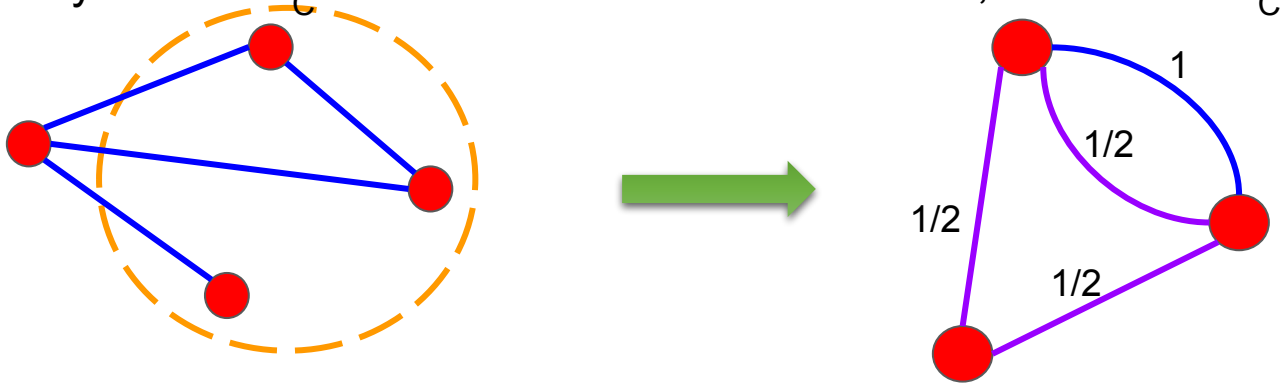

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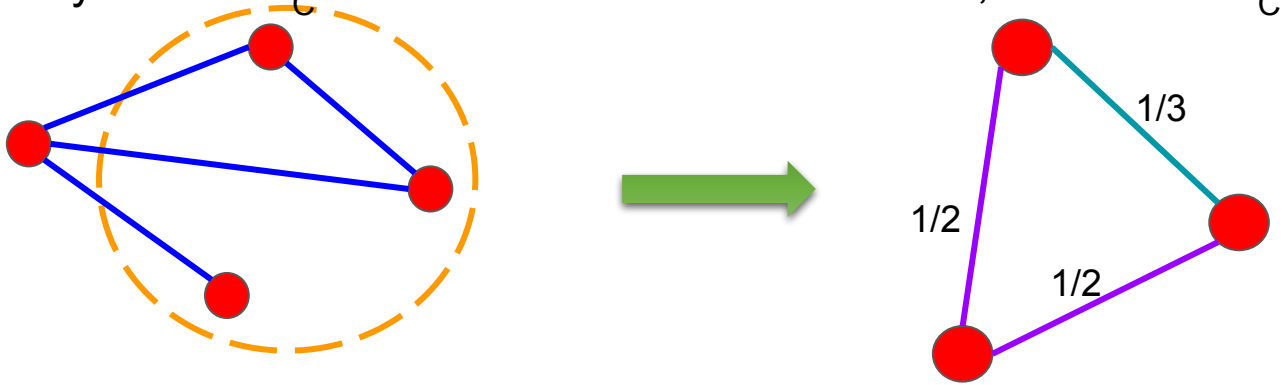

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$$L = \begin{bmatrix} L_{FF} & L_{FC} \\ L_{CF} & L_{CC} \end{bmatrix} \quad \text{and} \quad SC(L, C) := L_{CC} - L_{CF} L_{FF}^{-1} L_{FC}$$

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Essentially Gaussian elimination on blocks.

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Can show that SC(L, C) is a Laplacian matrix.

Cholesky factorization: $$L^{-1} = \begin{bmatrix} I & -L_{FF}^{-1} L_{FC} \\ 0 & I \end{bmatrix} \begin{bmatrix} L_{FF}^{-1} & 0 \\ 0 & SC(L, C)^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ L_{CF} L_{FF}^{-1} & I \end{bmatrix}$$

Dynamic Schur Complements by Random Walks

Approach: solve dynamic effective resistance by maintaining approximate dynamic SC (need approximate b/c the true SC is dense)

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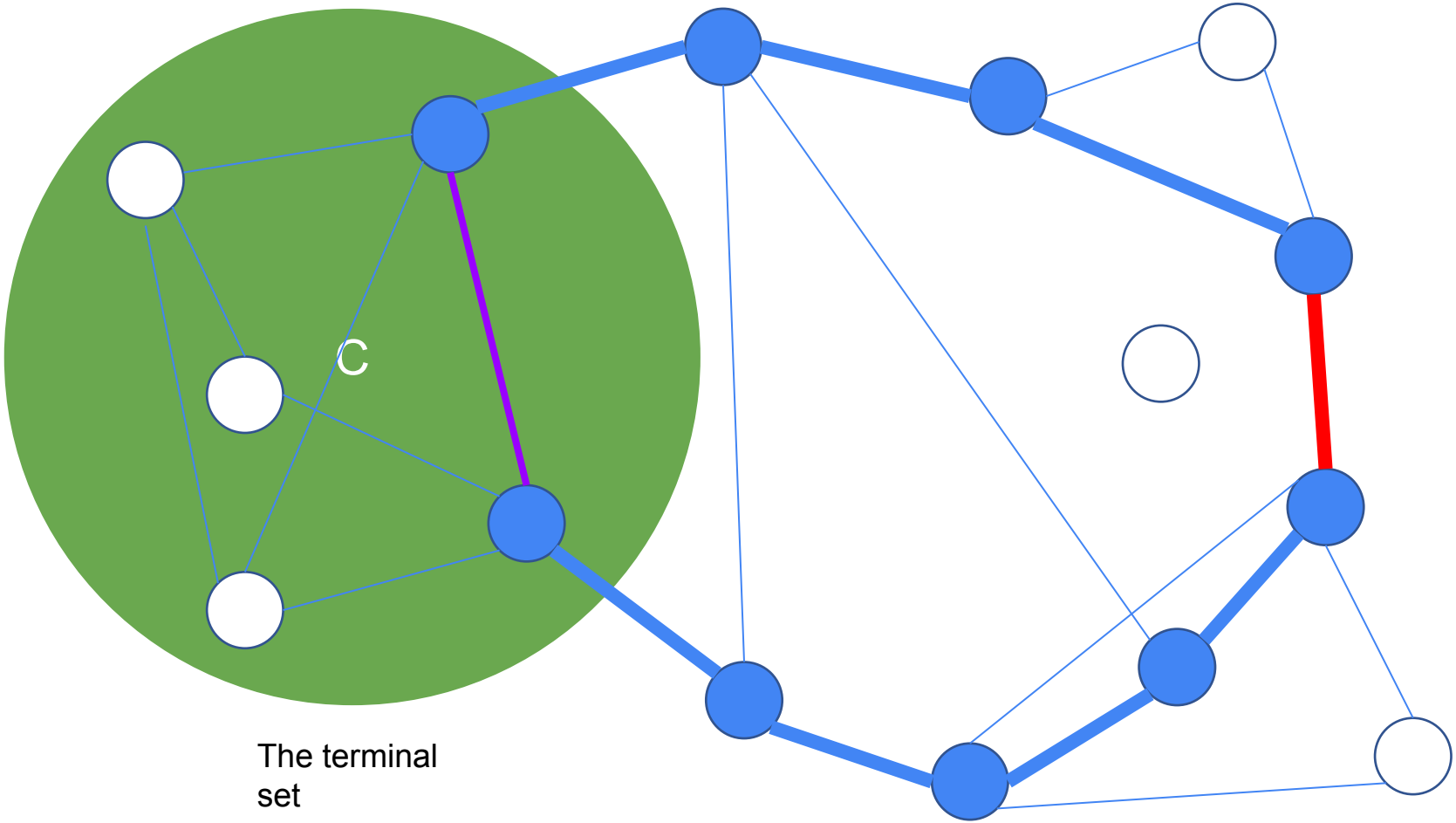
[DGGP19]: Maintain SC approximation by sampling random walks.

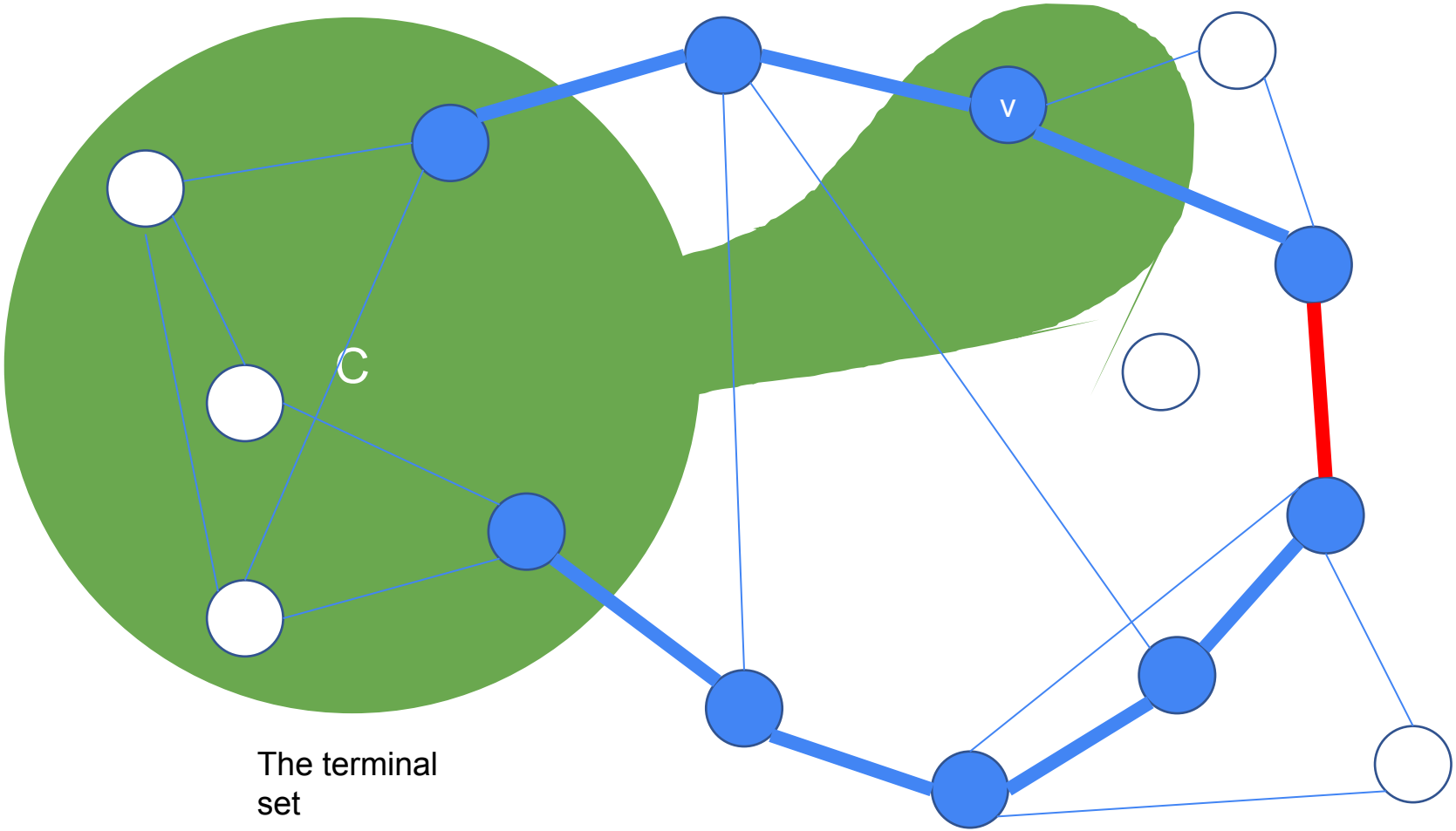
Theorem: Let C be a subset of vertices of a graph G = (V, E, r). Consider the following procedure. For each edge e = (u, v) in E, repeat p = $O(\varepsilon^{-2} \log n)$ times:

1. Run a random walk from u (resp. v) with exit probability proportional to r^{-1} until it hits C, say at t_1 and t_2.
2. Let W be the set of edges on the walk when connected using edge (u, v).
3. Add edge (t_1, t_2) to a graph H with resistance = $p \sum_{f \in W} r_f$

Then H is a (1+ε)-approximation spectrally of SC(L(G), C).



The terminal set



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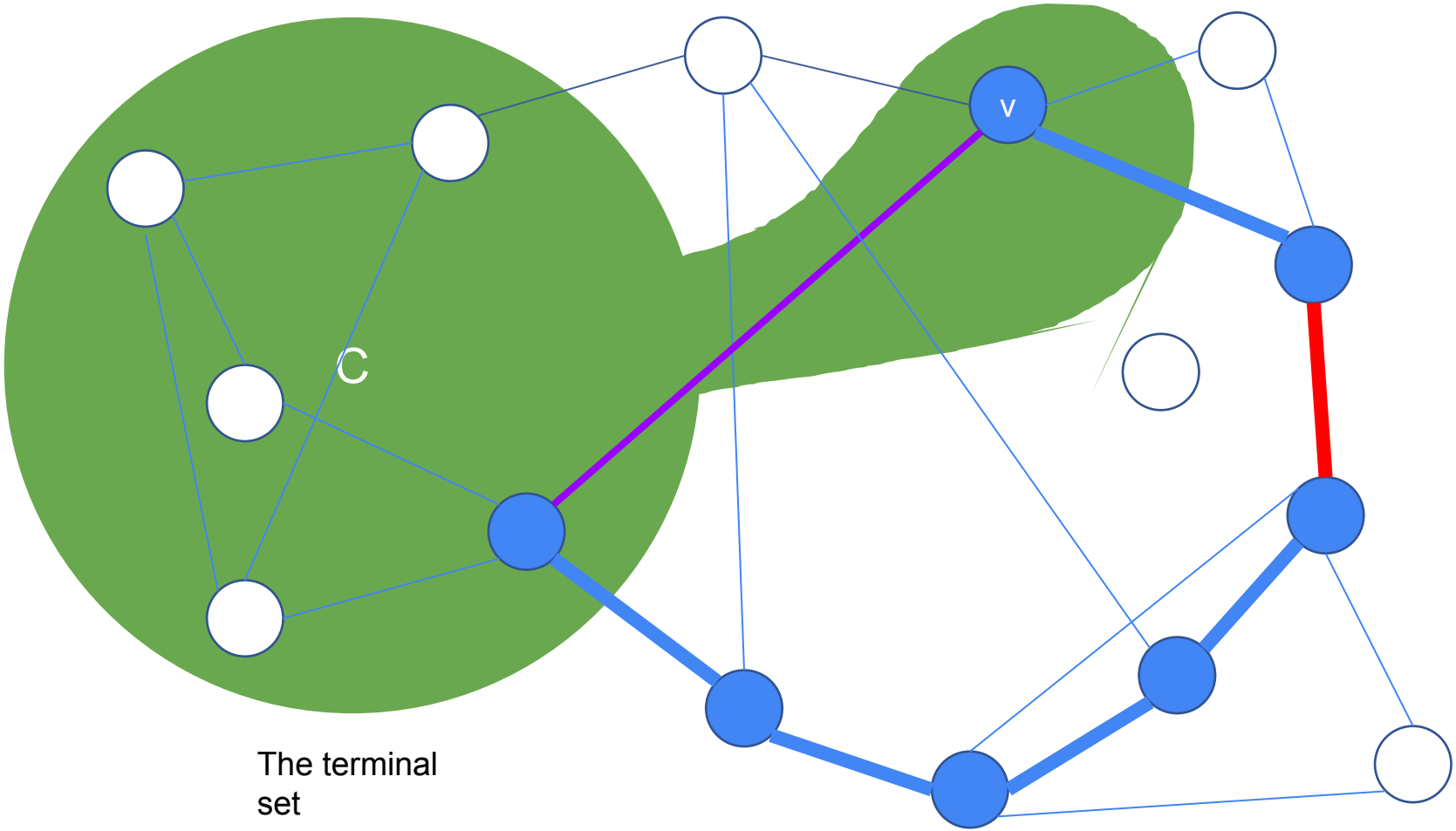

The terminal set

Dynamic Schur Complements by Random Walks

Algorithm for dynamic SC under resistance changes:

1. Initialize random walks from each edge e in G.
2. When edge e = (u, v) has resistance changed, add both endpoints u, v into the set C (the *terminals*). Shortcut random walks accordingly.
3. Update the resistance of edge e (which now fully inside C)

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When effective resistance of s-t is queried:

1. Add s, t to the set C (the terminals).
2. Return $\chi_{st}H^{-1}\chi_{st}$, where H is the approximation to SC(L, C) maintained.

Step 2 of this can take time $\tilde{O}(|C|\varepsilon^{-2})$ if we use a $1+\varepsilon$ sparsifier of H.

Electric Flow Concepts

For a demand vector d, we say that a flow f *routes* d if $B^T f = d$.

Electric *potentials* for demand d is given by $\Phi = L^{-1}d = (B^T R^{-1} B)^{-1} d$.

Electric flow: $f = R^{-1}B\Phi = R^{-1}B(B^T R^{-1} B)^{-1} d$.

Ohm's Law: for e = (u, v) we have $f_e = (\Phi_u - \Phi_v)/r_e$.

Formal Dynamic Electric Flow Setup

Problem: Given initial graph G = (V, E, r), source s, sink t, accuracy ε,

1. Update: change resistance of edge e.
2. Query: for the unit s-t electric flow $f = R^{-1}B(B^TR^{-1}B)^{-1}\chi_{st}$, guaranteed that the energy is <= 1, i.e. $\|R^{1/2}f\|_2^2 = \sum_{e\in E} r_e f_e^2 \leq 1$, return a subset S of edge that contains all edges e with energy $r_e f_e^2 \geq \varepsilon^2$, and $|S| = O(\varepsilon^{-2})$.

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Harder than dynamic ER because we have to report the high energy edges -- in dynamic ER we are told the pair of vertices in the ER query.

Reduction to Maintaining Energy of Subsets

We wish to find coordinates of the vector $R^{1/2}f$ that are >= ε, given $|R^{1/2}f|_2$ <= 1.

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L2 heavy hitter [KNPW11]: there is a sketch matrix Q that is $\tilde{O}(\varepsilon^{-2})$ x n, such that for any vector y in R^n, and vector v satisfying |v - Qy|_inf <= ε/100, we can find all coordinates of y that are >= ε in time $\tilde{O}(\varepsilon^{-2})$. Q has all entries in {-1, 0, 1}.

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Use this for $y = R^{1/2}f$ -- need to approximately maintain
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Demand projection

Potentials on terminal

$\tilde{O}(\varepsilon^{-2})$ length $|C|$ vectors

Length $|C|$ vector

Reduction to Maintaining Energy of Subsets

Maintain: $QR^{1/2}f$

Plan: approximately maintain both quantities dynamically.
We'll start by discussing the potentials, then the demand projection.

$$QR^{-1/2}B(B^TR^{-1}B)^{-1}\begin{bmatrix} I & 0 \\ -L_{CF}L_{FF}^{-1} & I \end{bmatrix}\chi_{st}$$

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Demand projection — Potentials on terminal

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Maintaining Potentials on Terminals

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Solution: Directly use the dynamic Schur complement from [DGGP19] for effective resistance!

Prove that (1+ε)-approximate Schur complement suffices for our purposes.

Maintaining Demand Projections

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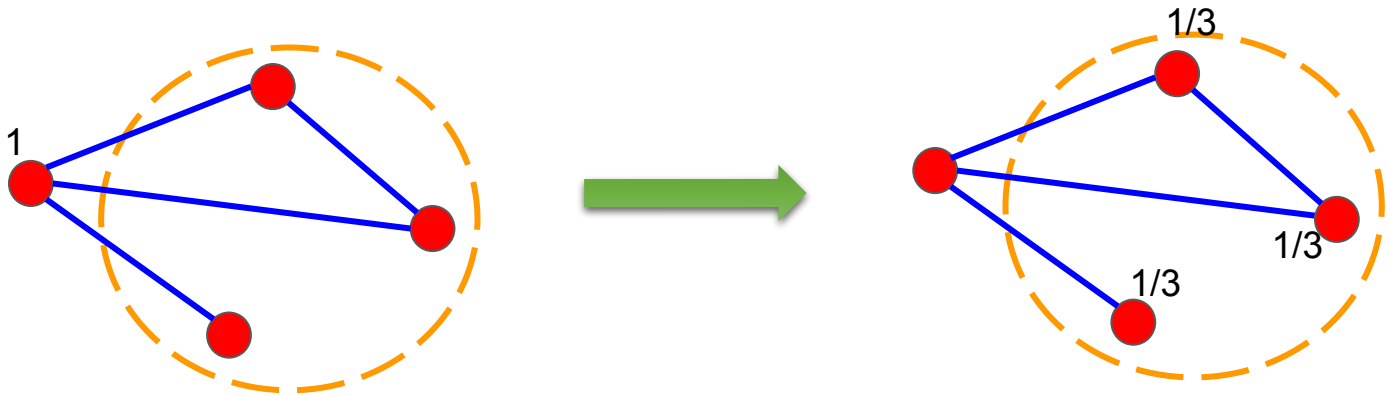

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So we assume that $d_v <= \deg(v)\varepsilon^{-1}$ from now on. Intuitively, think of as O(1).

Approach 1: Project Demands by Random Walks

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Issue: Say d_v = O(1), and we sample p walks from v until they hit C. “Variance” of this sample is O(1) per sample, so 1/p when you average over s walks.

There are m vertices / edges to sample from, so the sum of variance is m/p: huge!

Estimate *Change* in Projected Demands by Sampling

Maintain: $\begin{bmatrix} L_{CF}L_{FF}^{-1} & I \end{bmatrix} d$. From now on, we will denote as $\pi^C(d)$.

Read as “projection of d onto C”.

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Coordinate v of the vector.

Indicator vector of vertex v.

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Proof: Consider first sampling a random walk from a vertex u to {C U v}. Now, take all the mass at v and continue random walking it until it hits C. This is a valid simulation of random walking from u -> C only. Equating these two formulations gives the result.

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Intuitive analysis: Both terms have variance ~cong(v)/p, where cong(v) = # of random walks passing through v and p = oversampling. Much smaller error!

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Doable with a single Laplacian system solve.

Amortized time $\delta m \varepsilon^{-3}$, as there are $\tilde{O}(\varepsilon^{-2})$ test vectors.

Pick $\delta << \varepsilon$, and trade off parameters properly (increase oversampling parameter).

Adaptive Adversaries and Checker Data Structure

As described, previous algorithm’s randomness might affect future queries.

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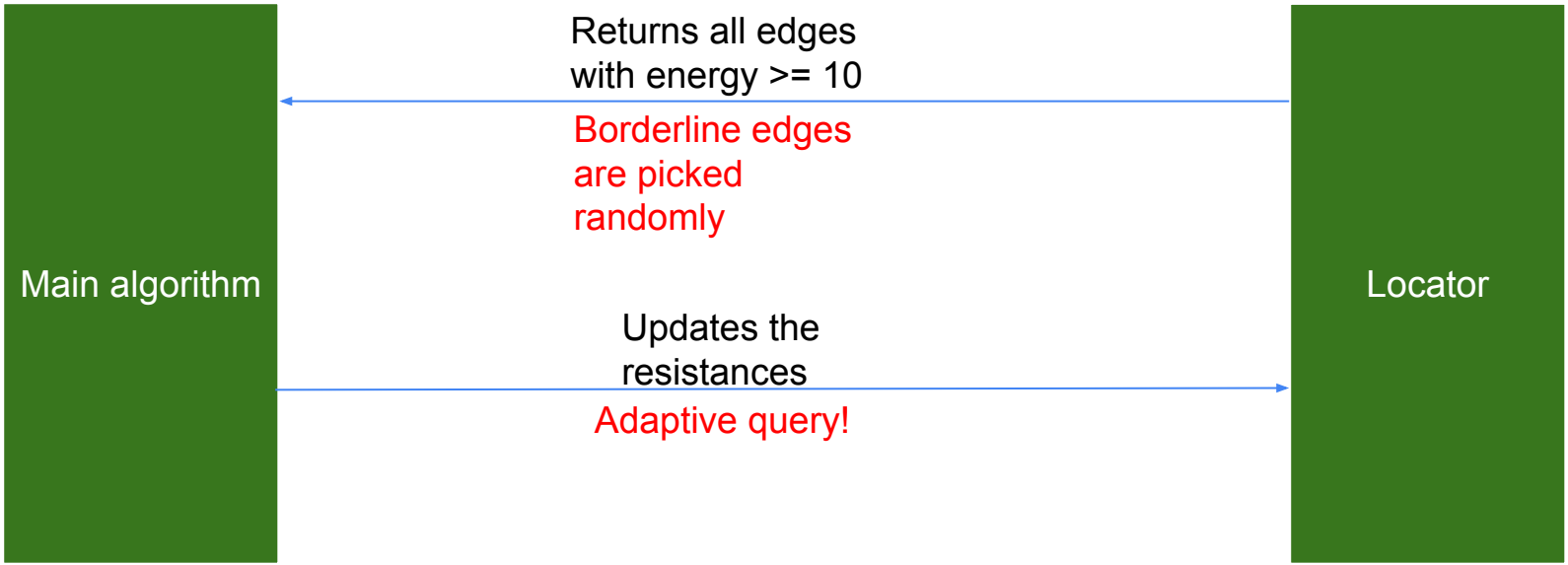
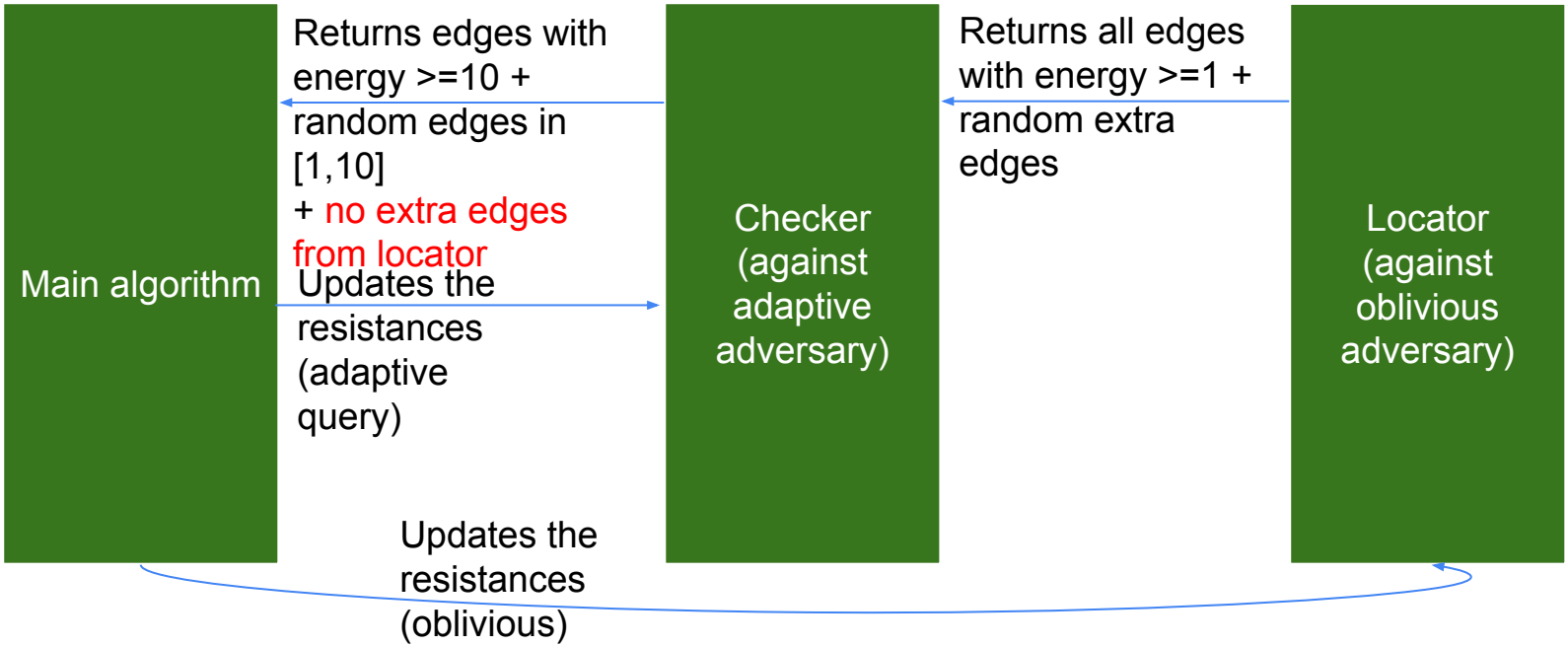
Need to carefully ensure that extra edges returned by the heavy hitter are not "accepted" by *Checker*, i.e. returned to the main IPM to update.

Also need to make sure extra edges returned by heavy hitter don't affect the responses of *Checker* on remaining edges.

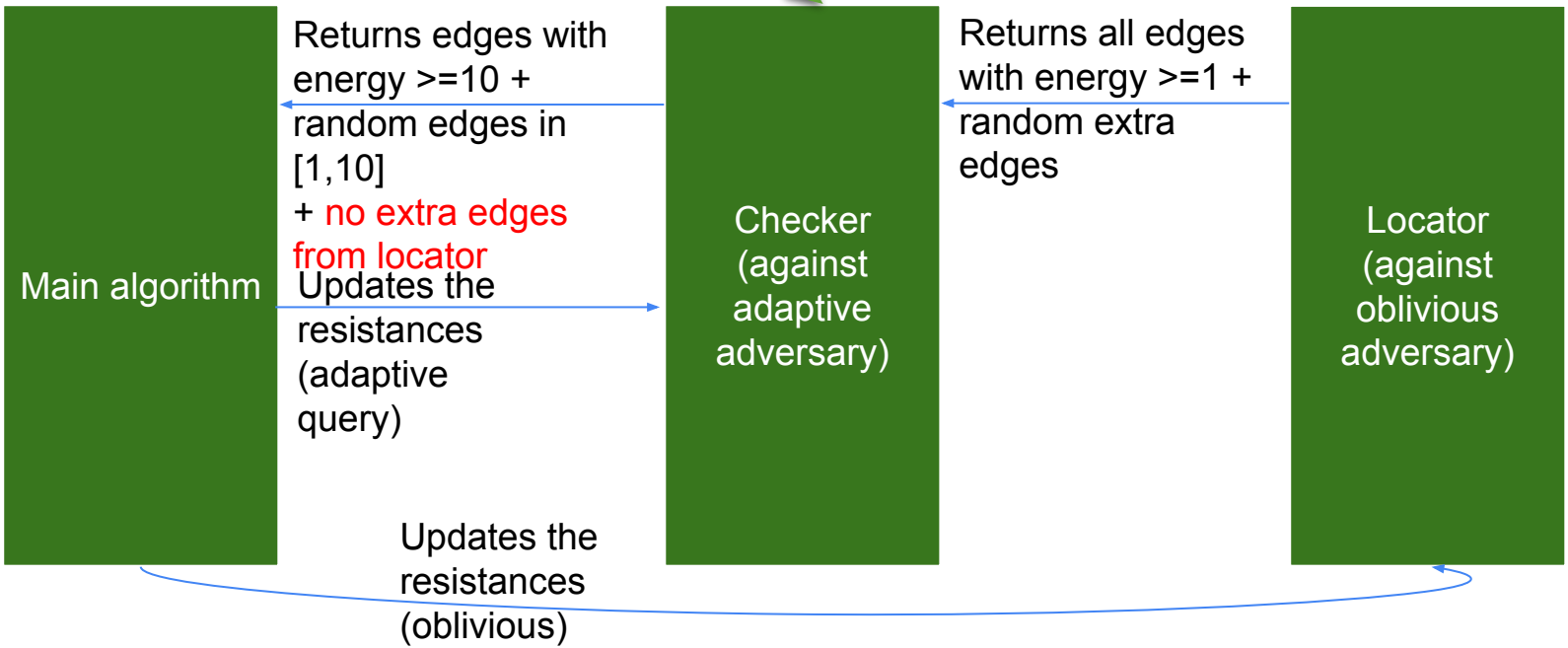

Main algorithm
Returns all edges
with energy >= 10
Borderline edges
are picked
randomly
Updates the
resistances
Adaptive query!
Locator

Adaptive *Checker*



Adaptive *Checker*

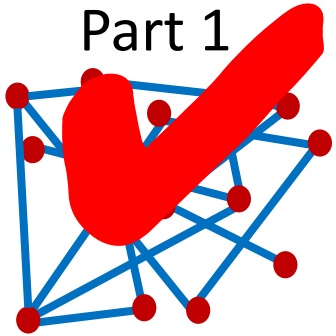
Make “semi-adaptive” by oversampling to capture all possible terminal sets / Schur complements possibly encountered.



Talk Organization

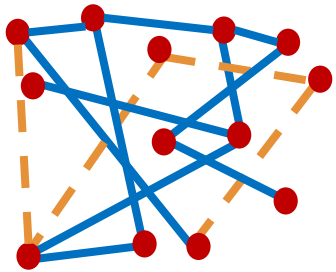
Flow Problems

Part 1



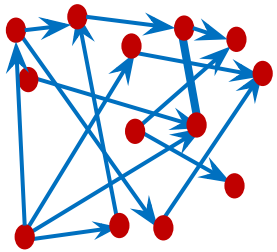
Fully Dynamic Electric Flows

Part 2



Putting it Together: Full Algorithm

Part 3



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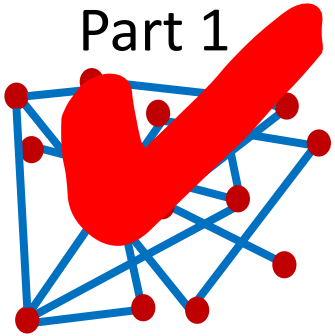
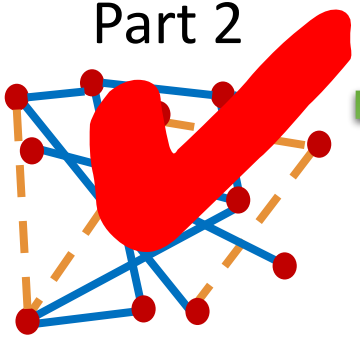


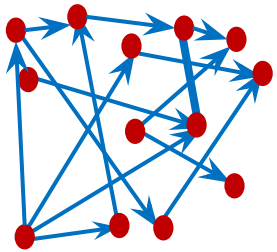
Fully Dynamic Electric Flows

Part 2



Putting it Together: Full Algorithm

Part 3



Full Algorithmic Picture

$k = m^{1/328}$. Goal is reduce residual flow by $(1-k/m^{1/2})$ in $\tilde{O}(m)$ amortized time.

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For $\tilde{O}(m^{1/2}/k)$ batched steps:

Split into k^4 equally sized steps of size $1/(k^3 m^{1/2})$.

Locator + *Checker* return edges S with >= ε fraction of flow in s-t electric flow.

Update resistances of edges in S.

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Update resistances of edges in S, pass to *Checker* and *Locator*.

Pay $\tilde{O}(m)$ time to bring flow back to “central path” of the interior point method.

Heuristic Analysis

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For $\tilde{O}(m^{1/2}/k)$ batched steps:

Amortized m^{1-c} per call / update.

Split into k^4 equally sized steps of size $1/(k^3 m^{1/2})$.

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Pay $\tilde{O}(m)$ time to bring flow back to "central path" of the interior point method.

Heuristic Analysis

Pay $\tilde{O}(m)$ after each "batched step" x $\tilde{O}(m^{1/2}/k) = \tilde{O}(m^{3/2}/k)$.

$O(k^{16})$ updates per phase x $\tilde{O}(m^{1/2}/k)$ phases x $O(m^{1-c})$ per update = $\tilde{O}(m^{3/2-c}k^{15})$.

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(log U) dependence comes from capacity scaling: our algorithm can handle arbitrary polynomial sized weights in $\tilde{O}(m^{3/2-1/328})$.

Additional Difficulties

Analysis of the interior point method is slightly non-standard, as we only use s-t electric flows during the method.

Standard IPM adds in electric circulations between steps to “recenter” a little.

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Standard IPM adds in electric circulations between steps to “recenter” a little.

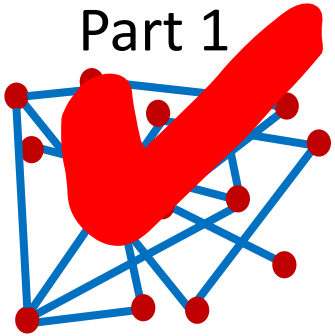
Some resistance updates accumulate over the course of several electric flow updates (instead of during a batched step).

Need to update such edges and trade off with cost of resistance updates in *Checker* and *Locator*.

Talk Organization

Flow Problems

Part 1



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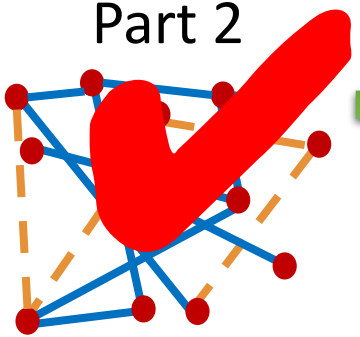

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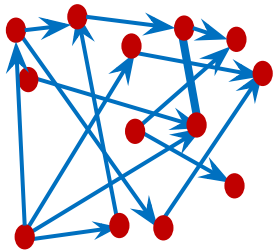

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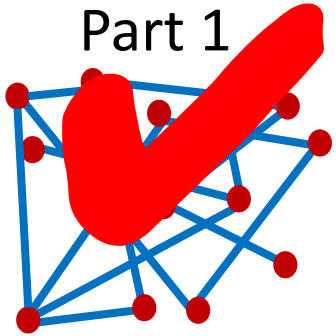

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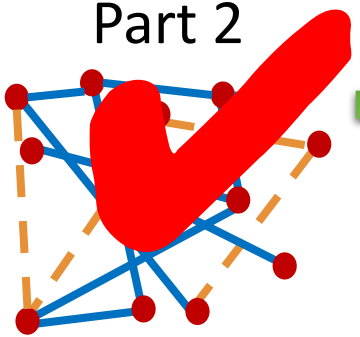

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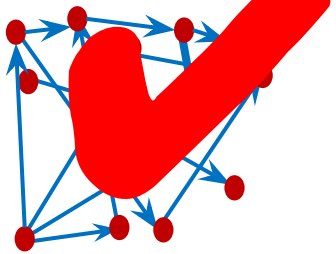

Conclusion

We give an algorithm that solves maxflow in $\tilde{O}(m^{1.497} \log U)$ on capacitated graphs

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First improvement to the $\tilde{O}(m^{1.5} \log U)$ algorithm of Goldberg-Rao 1998.

Approach is based on augmenting electrical flows and sublinear m^{1-c} algorithm for a specific instance of a dynamic electric flow problem.

Precisely, detect and return high energy edges in the s-t electric flow on a dynamic graph with changing resistances.

Further Directions / Open Problems

We believe that every piece of our algorithm can be improved or simplified.

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Adaptivity: Current solution involves using a Checker and oversampling to control *all possible* terminal sets. Resparsification? [**BBGNSSS20**]

Fully Dynamic Electrical Flows:
Sparse Maxflow Faster than Goldberg-Rao

arXiv: 2101.07233

The End

Questions?

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- website: *yangpliu.github.io*