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arXiv: 1907.10779

Constant Girth Approximation for Directed Graphs in Subquadratic Time

Shiri Chechik, Yang P. Liu, Omer Rotem, Aaron Sidford




Talk Outline

Part I:
Background

Talk Outline

Part I: Background → Part II: Algorithms

Approximation Algorithms for the Girth

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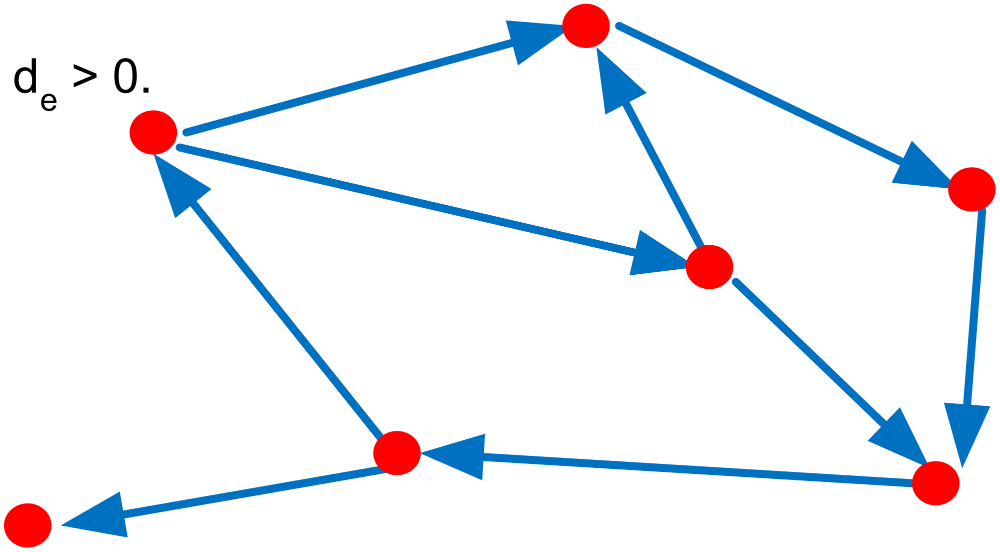
Directed Graph G = (V,E)

- n vertices, m edges
- Edge e = (u -> v) has length $d_e > 0$.



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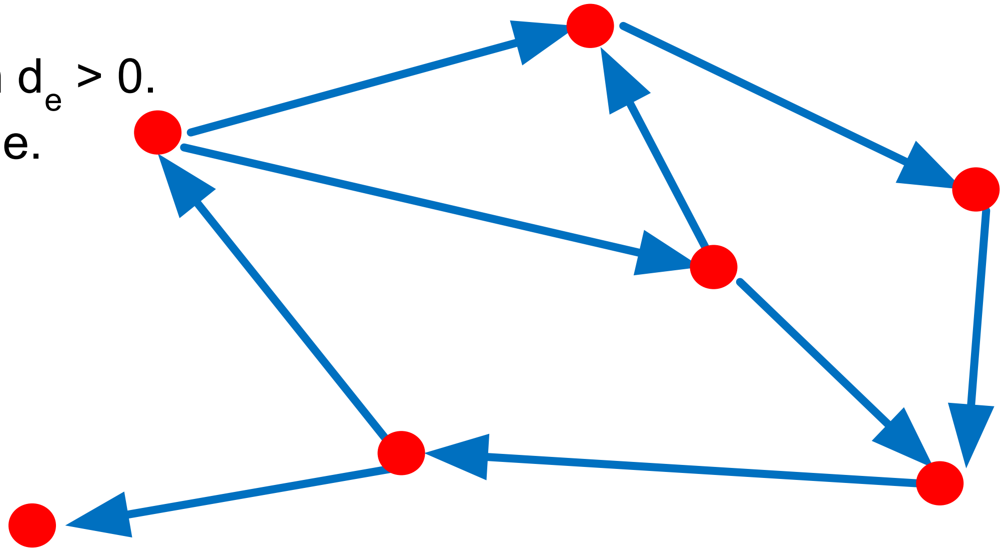

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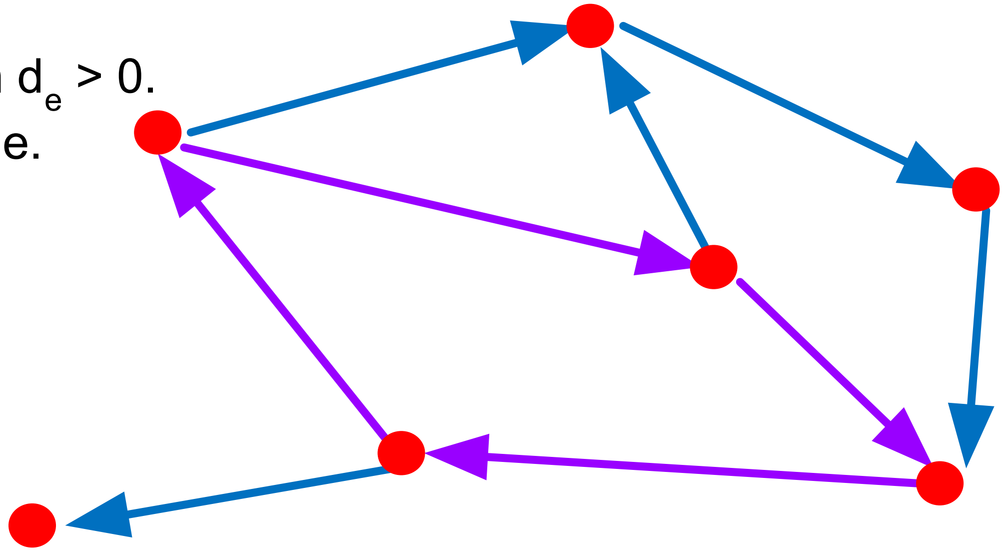

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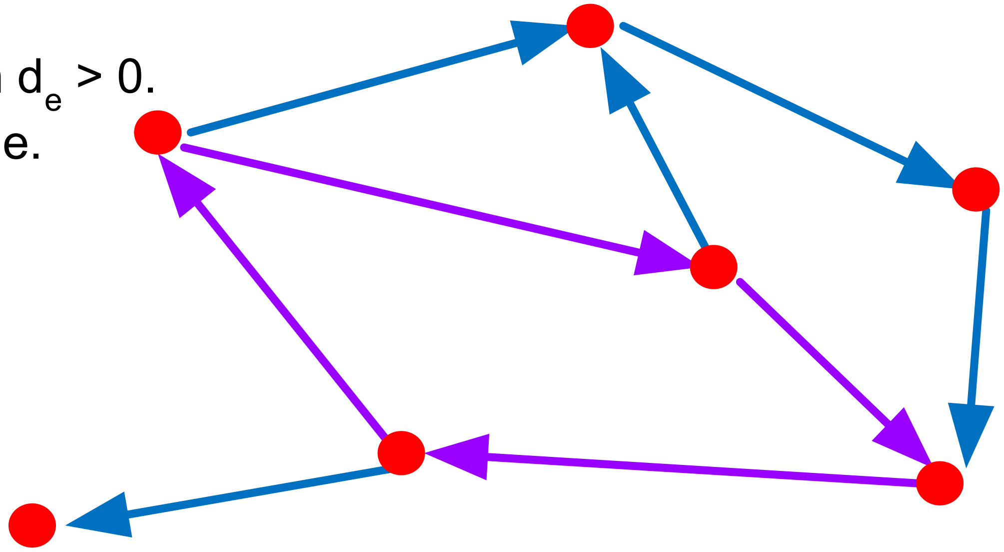

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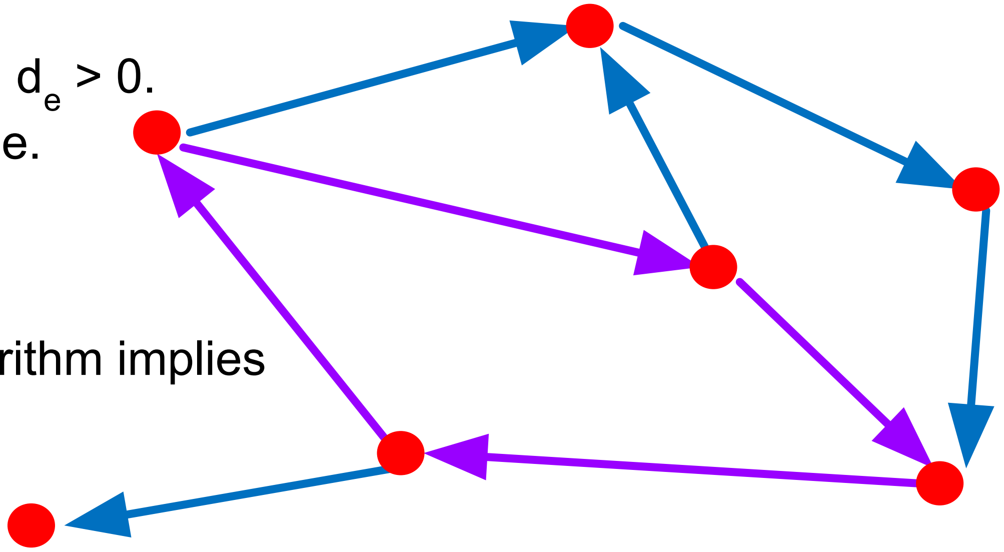

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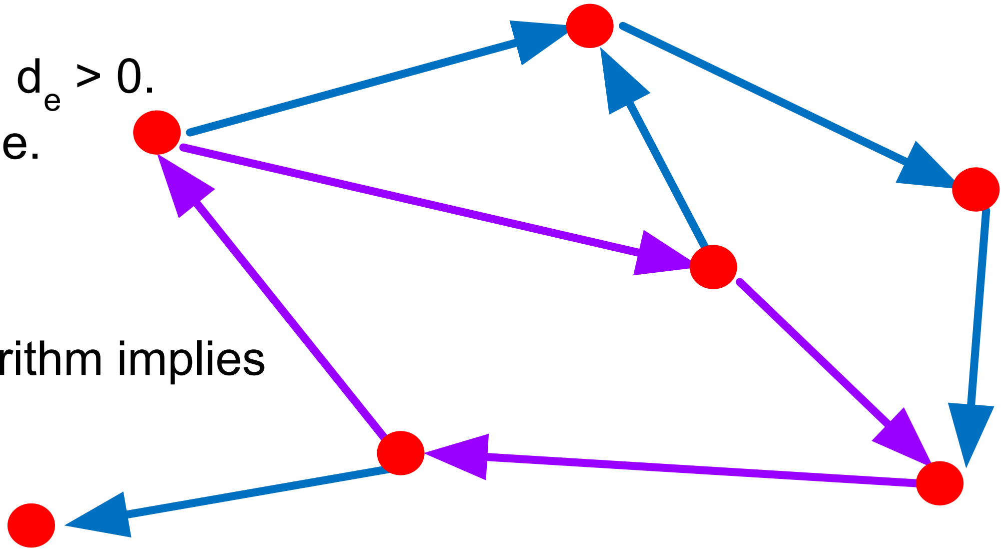

Approximation Algorithms

- [WW10] $n^{3-\varepsilon}$ time exact algorithm implies subcubic APSP.
- Focus on multiplicative approximation algorithms.

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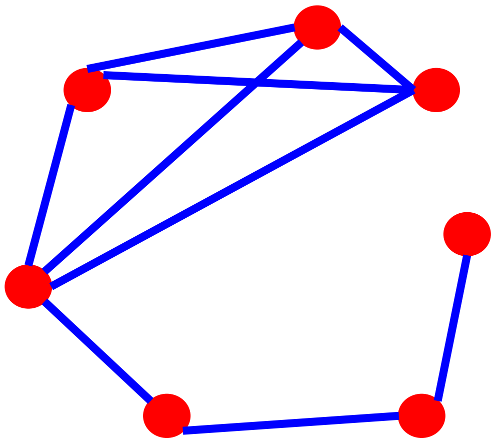

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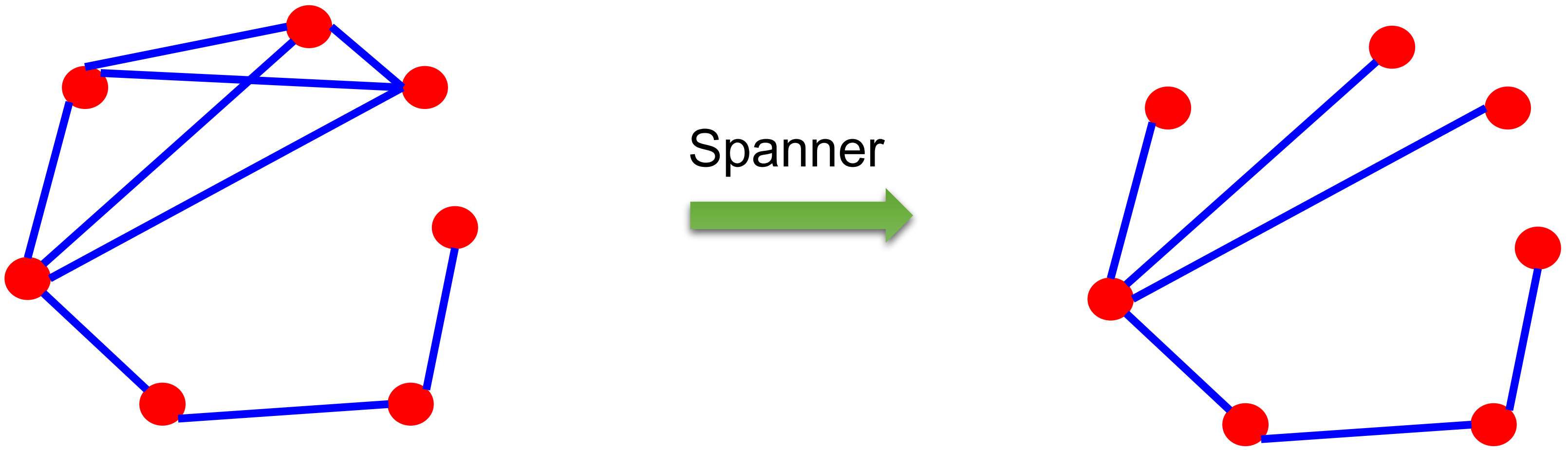

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[LL09,RW12,DKS17] -- best girth approximation algorithms
do not use spanners directly.

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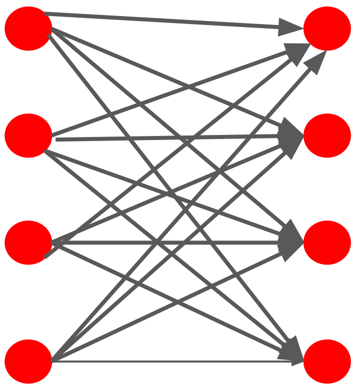

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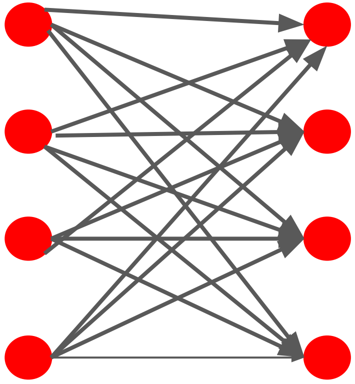

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Subgraph H with $d_G(u \leftrightarrow v) \leq d_H(u \leftrightarrow v) \leq c \cdot d_G(u \leftrightarrow v)$

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Requires computing full roundtrip metric

c-Roundtrip spanner:
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[RTZ08, CDG20] (2k-1)-roundtrip spanner exists with $O(kn^{1+1/k} \log(nW))$ edges, no efficient algorithm.

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[TZ05]	$(2k-1, mn^{1/k})$	Undirected
[RW12]	$(3/2, n^{5/3})$	Undirected
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Constant factor girth approximation in subquadratic time! In fact, exponent arbitrarily close to 1.

Directed Graph Primitives Matching Undirected Graphs

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1. Girth approximation
2. Laplacian solving
3. Parallel reachability / shortest paths
4. Maximum flow


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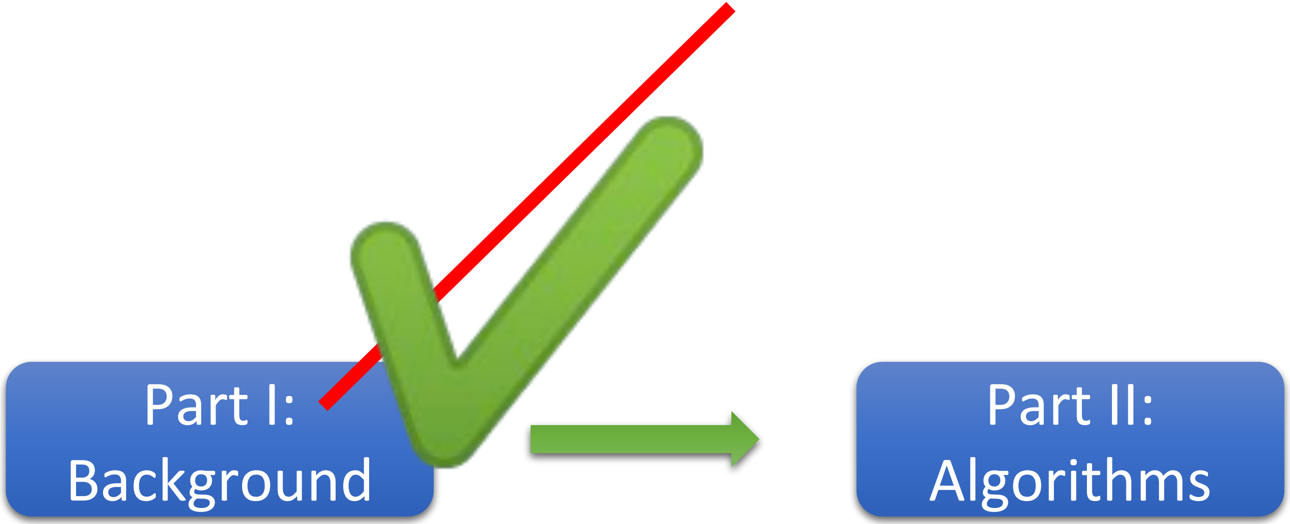
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Rest of Talk

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Algorithm: pick arbitrary v, find minimal d such that $|B(v,d+1)| \le n^{1/k}|B(v,d)|$.

Add spanning tree on B(v,d+1) and delete B(v,d).

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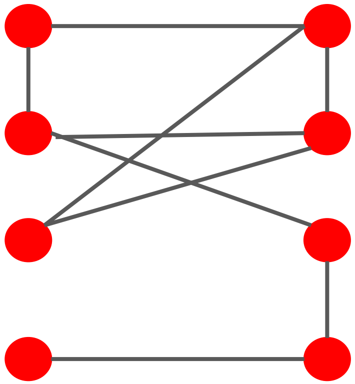

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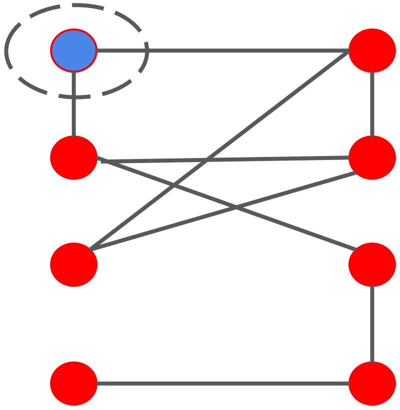

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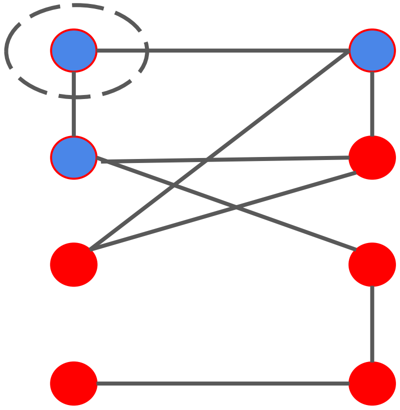

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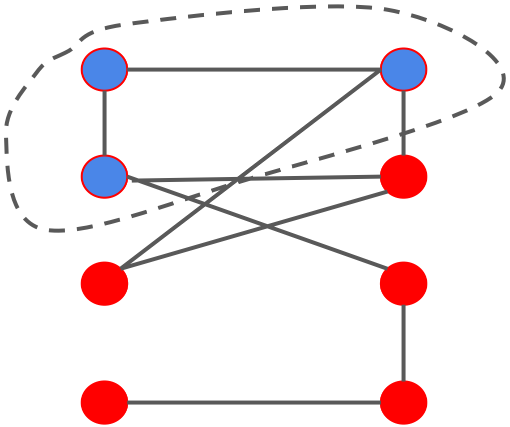

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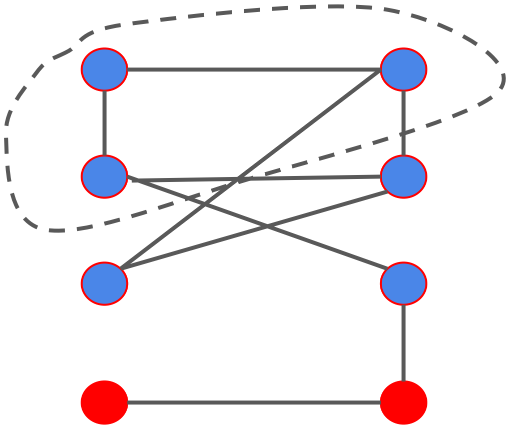

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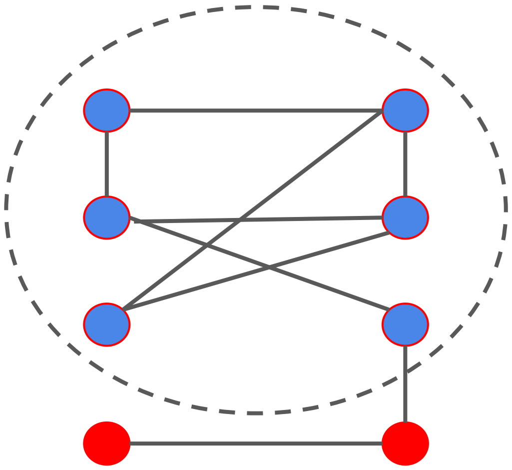

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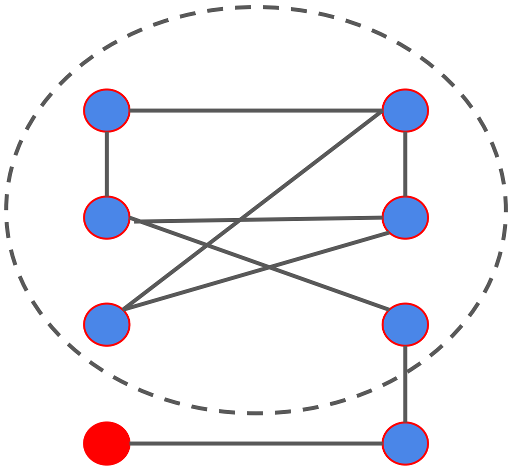

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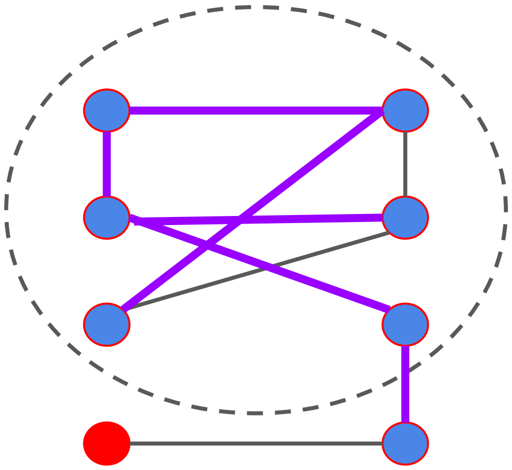

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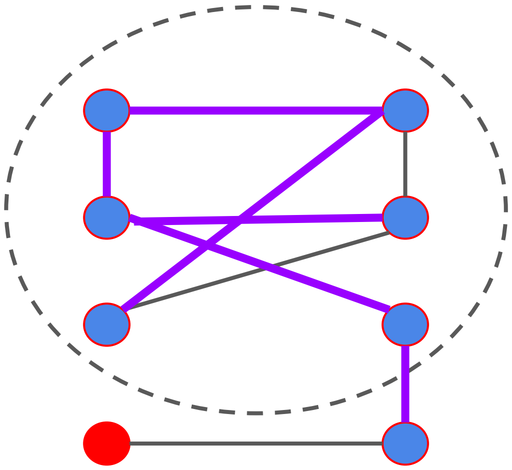

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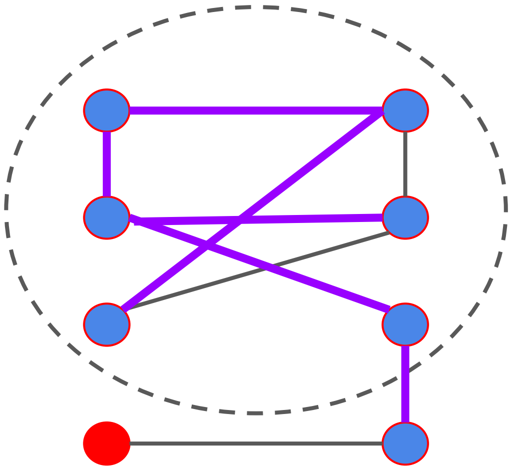

Analysis

(2k-1)-spanner:
$|B(v,d+1)| <= n^{1/k}|B(v,d)|$ will be violated for some $d <= k$.

At most $n^{1+1/k}$ edges:
charge $n^{1/k}$ edges per vertex deleted by the condition $|B(v,d+1)| <= n^{1/k}|B(v,d)|$.

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Potential solution: Recurse on the outball instead of just adding a spanning tree.

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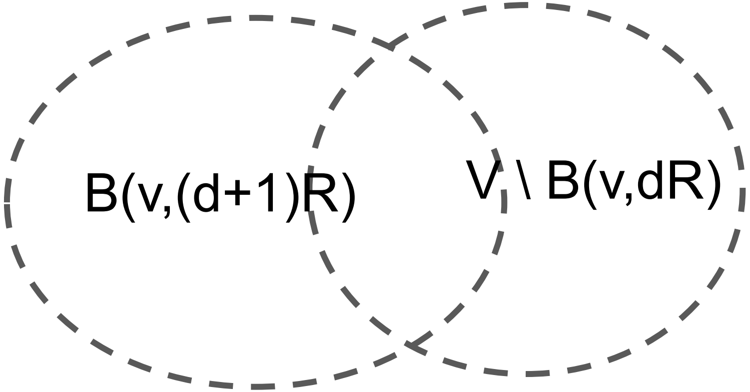
Recurse on **overlapping** pieces B(v,(d+1)R) and V \ B(v,dR).



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Theorem [CLRS20]: ***Deterministic*** **O(k loglog n) girth approximation in $O(mn^{1/k})$ time.**

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Intuitively, use Observation 1 and random sampling to reduce to “checking” $n^{1/2}$ vertices per vertex v.

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3. For $1 <= i <= M$ and each v in V(G)
4. $T_i(v) = \{s \text{ in } S_i : d(v,s) <= R \text{ and } d(s,t) <= 2R \text{ for all } t \text{ in } R_1(v),\ldots,R_{i-1}(v)\}$.
5. $R_i(v)$ is a random sample of $T_i(v)$ of size 100 log n.

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Intuitively, $T_i(v)$ is subset of S_i that algorithm still thinks can be in a cycle of length R with v.

Algorithm 2: Random Sampling and Distance Tests

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1. Sample M sets S_1, S_2, ... , S_M of size $O(n^{1/2}\text{polylog}(n))$.
2. Run Dijkstra for shortest paths to / from all vertices in S_i for 1 <= i <= M.
3. For 1 <= i <= M and each v in V(G)
4. $T_i(v)$ = {s in S_i : d(v,s) <= R and d(s,t) <= 2R for all t in $R_1(v),\ldots,R_{i-1}(v)$}.
5. $R_i(v)$ is a random sample of $T_i(v)$ of size 100 log n.
6. If $|T_M(v)|$ <= 100 log n, ball grow from v.

Intuitively, $T_i(v)$ is subset of S_i that algorithm still thinks can be in a cycle of length R with v.

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Dijkstra to/from S_i take $O(mn^{1/2}$ polylog(n)) total time.

Can show $|T_i(v)| <= .9\ |T_{i-1}(v)|$ with high probability or find cycle of length 4R.

If $|T_i(v)| <= O(\log n)$ then ball growing from v only visits $O(n^{1/2})$ vertices.

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Result: With high probability, $O(mn^{1/2})$ runtime and 4-approximation.

Get 3-approximation by being more careful.

Algorithm 3: Combination of Algo 1 and 2

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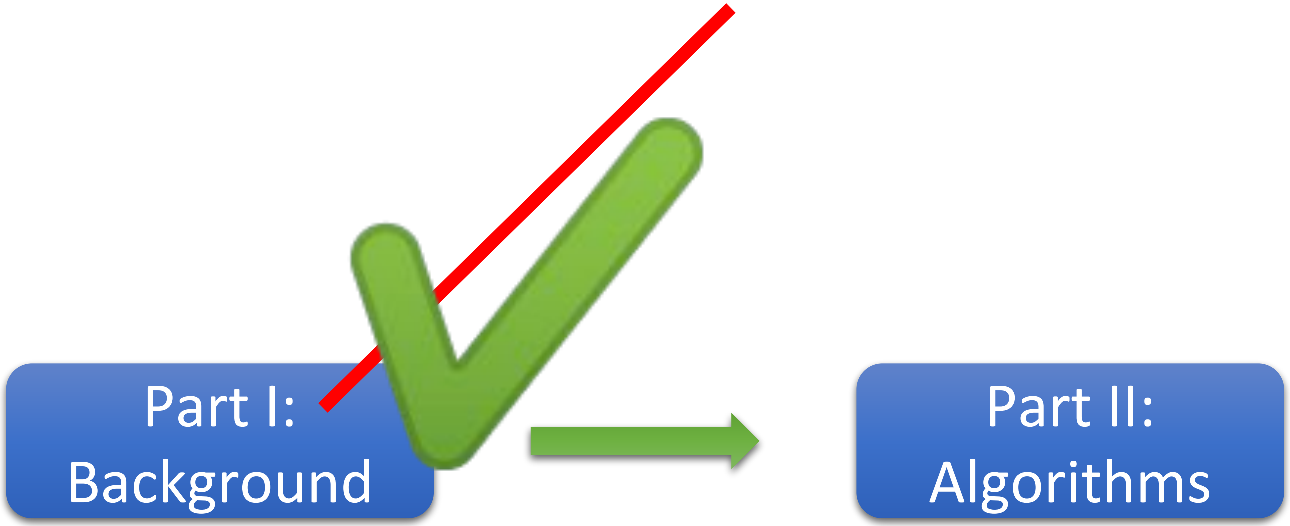
Result: $O(k \log k)$ girth approximation in $mn^{1/k}$ time.


Part I:
Background
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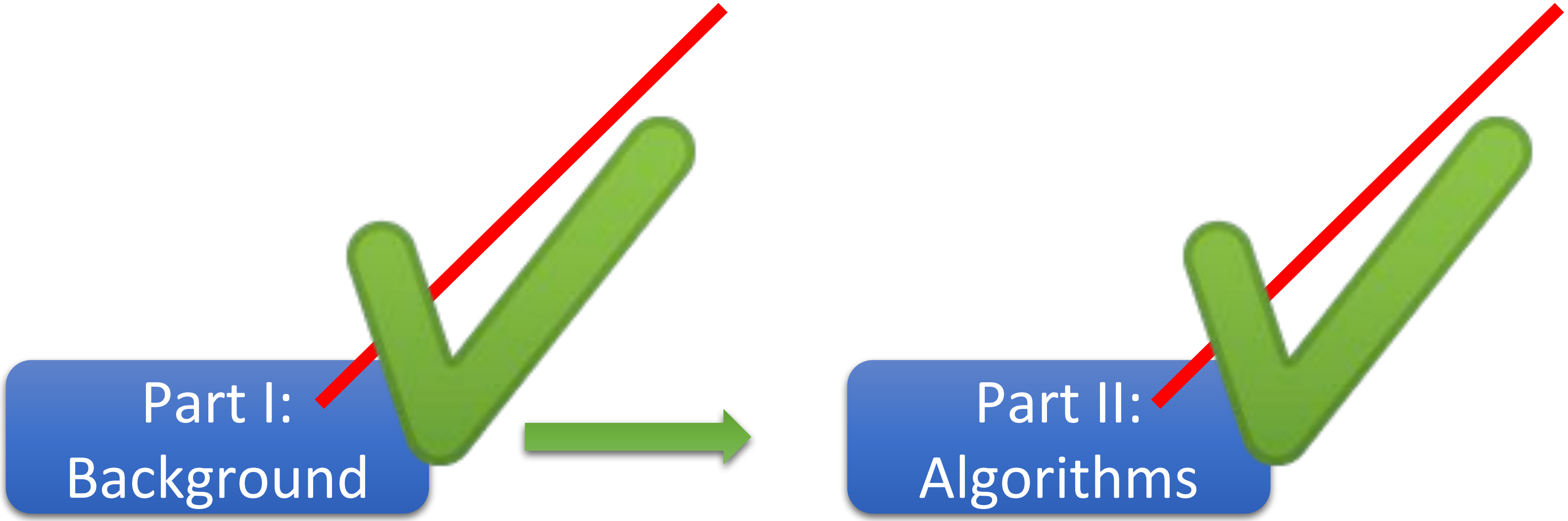
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Recent Work [DW20]: 2-approximation in subquadratic time. Significantly improved constants in the O(k log k) approximation, eg. 4-approximation in $mn^{.414}$.

Constant Girth Approximation for Directed Graphs in Subquadratic Time

arXiv: 1907.10779

The End

Questions?

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